Assignment 1

is on website

Due Mon Sept 28, 8:30 am

Submit pdf or jpg files to D2L Dropbox

Test 1

5 Questions, 50 minutes

Covers 11.7, 12.1–12.7

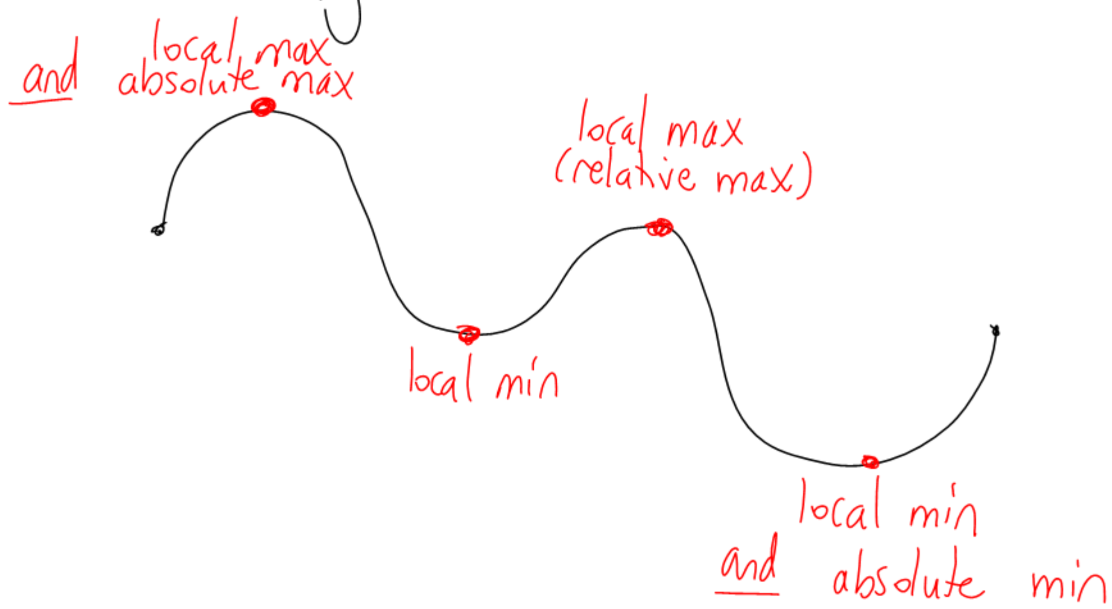
Section X02 Thurs Oct 1st

Bring a calculator and music/earplugs

Practice Problems on website

12.5 Multivariable Optimization

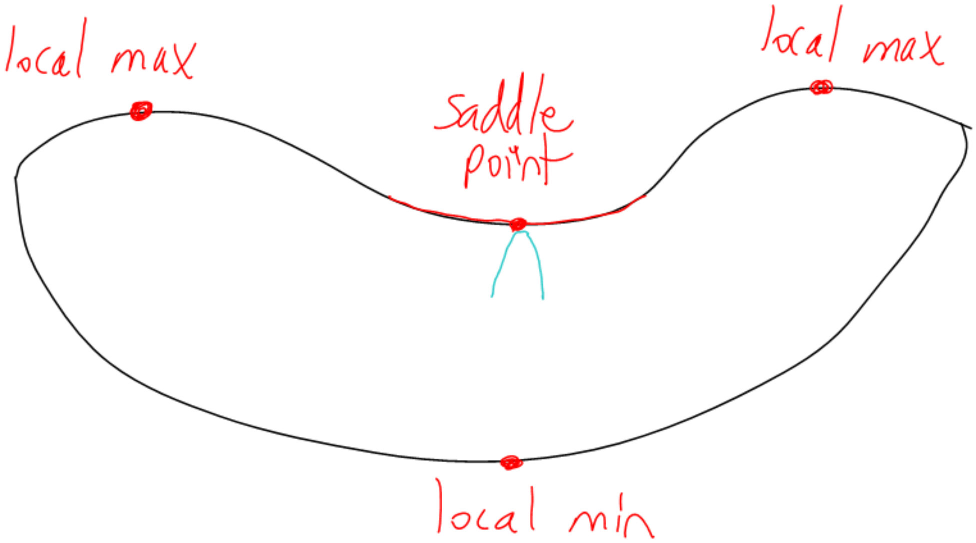
From single-variable calculus:



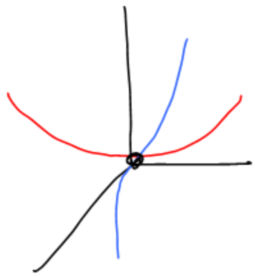
Def

Points where z_x and z_y are both 0 or undefined are called <u>critical points</u>.

- Could have $z_x = 0$ and z_y undefined, or vice versa, or $z_x = 0 = z_y$, or z_x and z_y both undefined.
- Critical points can be a local max, or a local min, or a saddle point, or none of the above.



Weirder critical point:



not a local max
not a local min
not a saddle point

<u>Ex</u>: Find all critical points of:

$$f = x^3 + xy^2 + 3x^2 - 3y^2$$

$$f_x = 3x^2 + y^2 + 6x$$
$$f_y = 2xy - 6y$$
both 0 or undefined

$$3x^2 + y^2 + 6x = 0 \quad (1)$$
$$2xy - 6y = 0 \quad (2)$$

(simpler equation) (2) : $2xy - 6y = 0$

$$2y(x-3) = 0$$
$$y = 0, \quad x = 3$$

Case 1: $y=0$	Case 2: $x=3$
$y=0 \rightarrow (1)$: $3x^2 + 6x = 0$	$x=3 \rightarrow (1)$: $27 + y^2 + 18 = 0$
$3x(x+2) = 0$	$y^2 = -45$
$x = 0, -2$	No solution
$(x,y) = (0,0), (-2,0)$	

$$(x,y) = (0,0), (-2,0)$$

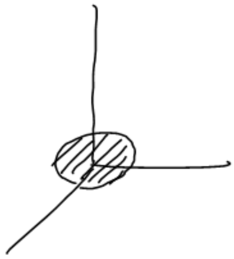
or $(x,y,z) = (0,0,0), (-2,0,4)$

$$f = x^3 + xy^2 + 3x^2 - 3y^2$$

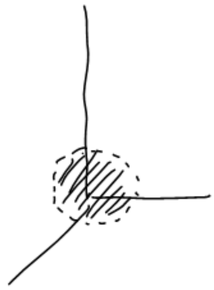
FACT

A continuous function defined over a closed domain attains an absolute maximum and an absolute minimum.

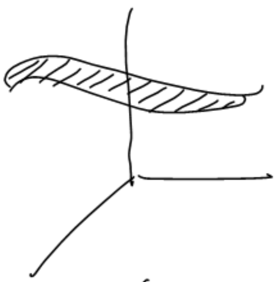
region of the xy-plane that includes its boundary



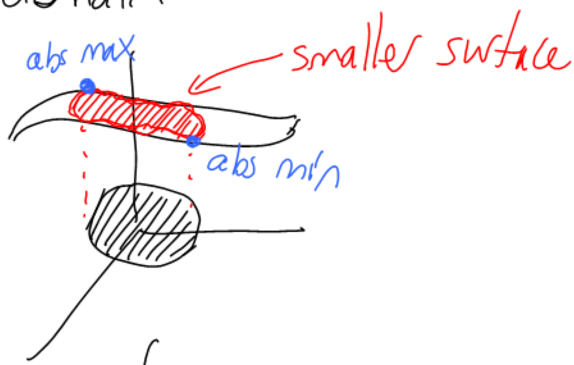
closed domain



non-closed domain

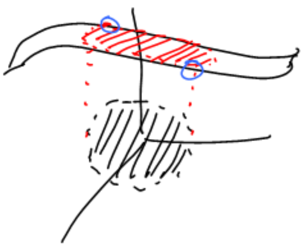


surface



surface over a closed domain

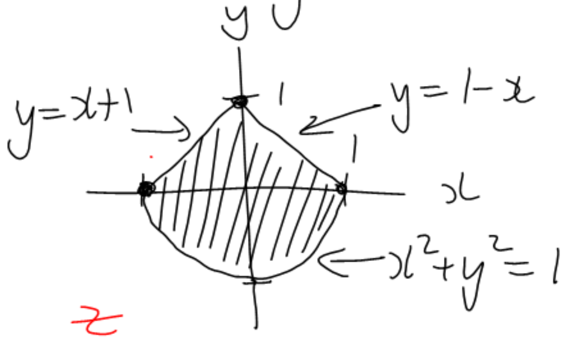
Can fail if the domain is not closed:

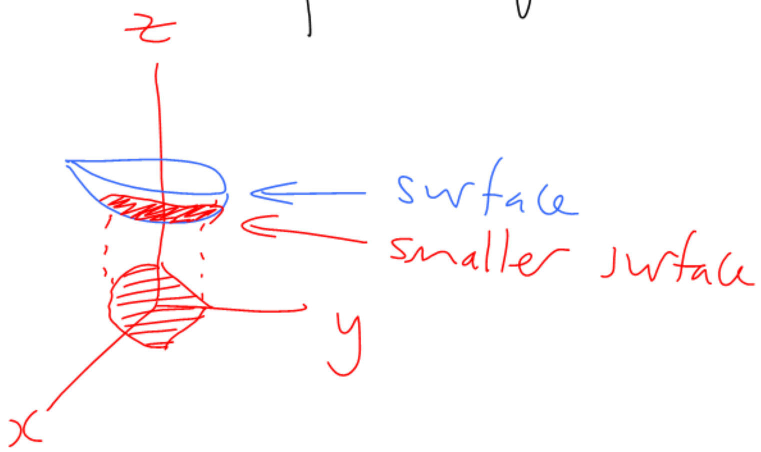


FACT

The absolute maximum and absolute minimum of f may occur in the interior of the domain, at a corner of the domain, or on a side of the domain.

<u>Ex</u>: Find the absolute maximum and absolute minimum of $f = x + x^2 + y^2$ over the region below:





1) Interior Critical Points

$$f = x + x^2 + y^2$$

$$\left.\begin{aligned} f_x &= 1+2x \\ f_y &= 2y \end{aligned}\right\} \text{both } 0 \text{ or undefined}$$

$1+2x=0 \Rightarrow x=-\frac{1}{2}$

$2y=0 \Rightarrow y=0$

$(x,y)=(-\frac{1}{2},0)$ in the interior of the domain ✓

2) Corners of the Domain

$(-1,0)$, $(0,1)$, $(1,0)$

3) Critical Points on Side 1: $y=x+1$

$y=x+1 \rightarrow$ $f=x+x^2+y^2$

$f=x+x^2+(x+1)^2$

$f'=1+2x+2(x+1)$

Set $f'=0$: $1+2x+2x+2=0$

$3+4x=0$

$4x=-3$

$x=-\frac{3}{4}$

$(x,y)=(-\frac{3}{4},\frac{1}{4})$

$y=x+1$

4) Critical Points on Side 2: $y=1-x$

$y=1-x \rightarrow$ $f=x+x^2+y^2$

$f=x+x^2+(1-x)^2$

$$f' = 1 + 2x + \underset{(-2)}{\cancel{2}}(1-x)\cancel{(-1)}$$

Set $f' = 0$: $\quad 1 + 2x - 2 + 2x = 0$

$$4x - 1 = 0$$

$$4x = 1$$

$$x = \frac{1}{4}$$

$$(x, y) = \left(\frac{1}{4}, \frac{3}{4}\right)$$

$y = 1 - x$

5) Critical Points on Side 3: $x^2 + {}^2 = 1$

$$f = x + \underbrace{x^2 + y^2}_{1}$$

$$f = x + 1$$

$$f' = 1$$

Set $f' = 0$: No solution

Points	$f = x + x^2 + y^2$	
$(-\frac{1}{2}, 0)$	$-\frac{1}{4}$	$\leftarrow$ abs min
$(-1, 0)$	0	
$(0, 1)$	1	
$(1, 0)$	2	$\leftarrow$ abs max
$(-\frac{3}{4}, \frac{1}{4})$	$-\frac{1}{8}$	
$(\frac{1}{4}, \frac{3}{4})$	$\frac{7}{8}$	

The absolute maximum is 2, achieved at $(x,y) = (1,0)$.

The absolute minimum is $-\frac{1}{4}$, achieved at $(x,y) = (-\frac{1}{2}, 0)$.