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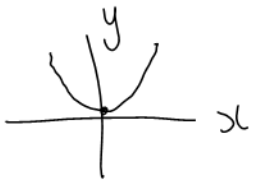
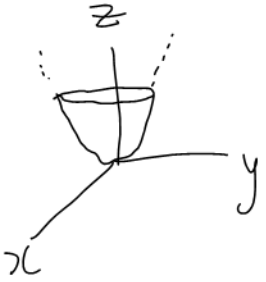
→ Math 250B

Overview of Multivariable Calculus

Single-variable function $y = x^2$



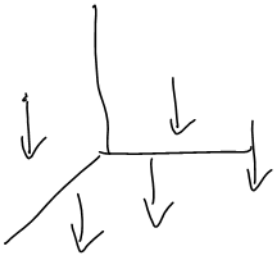
multi-variable function $z = x^2 + y^2$



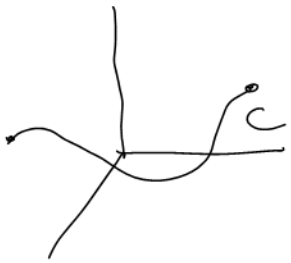
I Differentiation
Rates of change
Maximize/minimize a function

II Integration
Calculate volumes under surfaces.

III Vector Fields



force field



Work done by the force field on a particle travelling along curve C?

11.7/12.1/12.2 Surfaces and Functions of Several Variables

Function of several variables: Input is 2 or more variables

Notation:

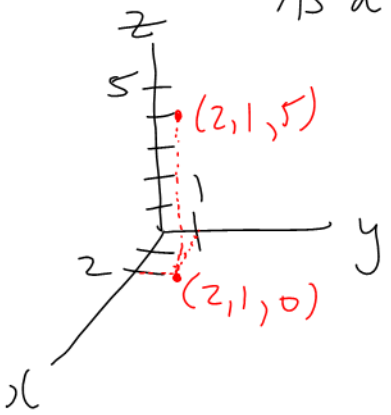
$$f(x,y) = x^2+y^2$$

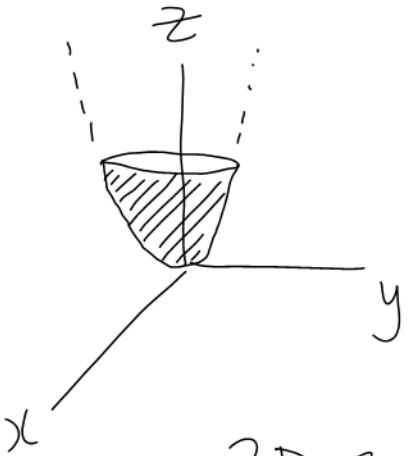
or $$f = x^2+y^2$$

or $$z = x^2+y^2$$

Evaluating: $f(2,1) = 5$

As a point: $(x,y,z) = (2,1,5)$



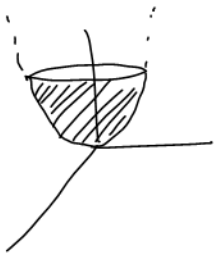


$z = x^2+y^2$

2D surface in 3D space

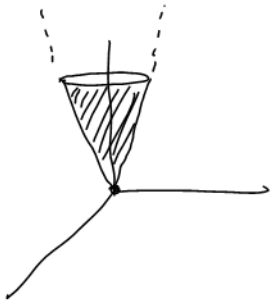
Five Important Surfaces

$z = x^2 + y^2$



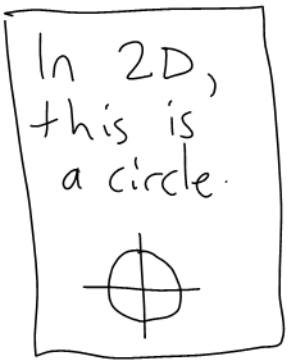
Paraboloid

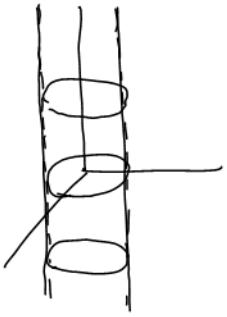
$z = \sqrt{x^2 + y^2}$



Cone

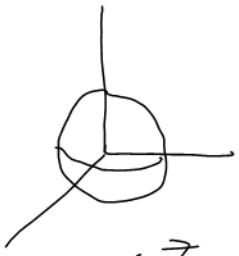
$x^2 + y^2 = 1$





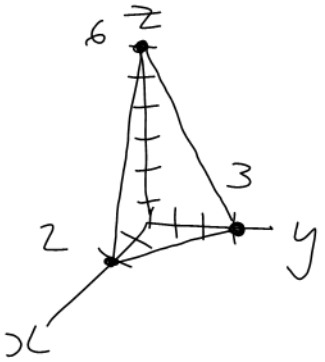
(Circular) Cylinder

$x^2 + y^2 + z^2 = 1$



Sphere

$3x + 2y + z = 6$



Plane (infinite surface)

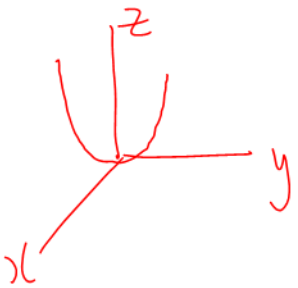
Graphing Paraboloids

Trace: Surface ∩ Plane, especially when the plane is $x=0$, $y=0$ or $z=0$.

Ex: Graph $z=x^2+y^2$ using traces.

$x=0$:

$z=y^2$



$y=0$:

$z=x^2$

$z=0$:

$0=x^2+y^2$

circle of radius 0

$(x,y)=(0,0)$

$z=x^2+y^2$

paraboloid

Contour Curve: Surface ∩ $(z=c)$

Ex: Graph $z=x^2+y^2$ using Contour curves.

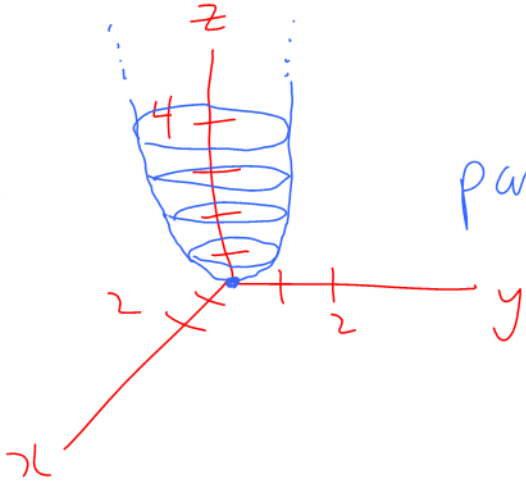
$z=0$: $0=x^2+y^2$ circle of radius 0 (centred at the origin)

$z=1$: $1=x^2+y^2$ " 1

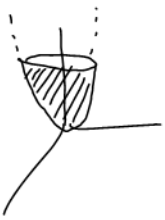
$z=4$: $4=x^2+y^2$ " 2

$z<0$: no real solution

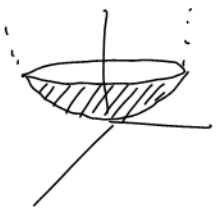
$z=x^2+y^2$ paraboloid



Ex: Transformations of $z=x^2+y^2$



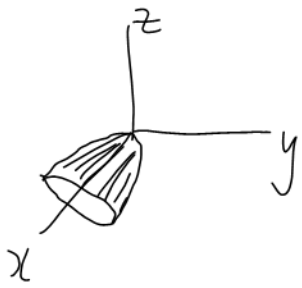
$z=\frac{1}{2}(x^2+y^2)$



$z=-(x^2+y^2)$

$z=x^2+y^2+1$

$x = y^2 + z^2$



Graphing Spheres

FACT $x^2 + y^2 + z^2 = R^2$ is a sphere of radius R centred at $(0,0,0)$.

Ex: Graph $z = \sqrt{1 - x^2 - y^2}$ using contour curves.

$z < 0$: no real solution

$z = 0$: $0 = \sqrt{1 - x^2 - y^2}$

$0 = 1 - x^2 - y^2$

$x^2 + y^2 = 1$ circle of radius 1

$z = 1$: $1 = \sqrt{1 - x^2 - y^2}$

$1 = 1 - x^2 - y^2$

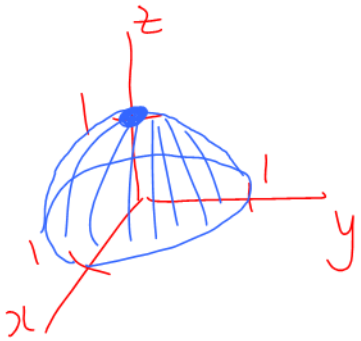
$x^2 + y^2 = 0$ circle of radius 0

$z = 2$: $2 = \sqrt{1 - x^2 - y^2}$

$4 = 1 - x^2 - y^2$

$x^2 + y^2 = -3$ no real solution

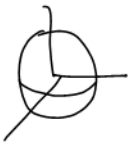
$z = \sqrt{1-x^2-y^2}$



upper hemisphere

Aside:

$z = -\sqrt{1-x^2-y^2}$ lower hemisphere 

$z = \pm\sqrt{1-x^2-y^2}$ sphere 

or $x^2+y^2+z^2=1$