

Assignment 1

is on website

Due Mon Sept 28, 8:30am

Submit pdf or jpg files to the D2L Dropbox

Test 1

6 questions, with parts

50 minutes

Fri Oct 2

Covers Ch 23

Bring calculator and music/earplugs

Practice Problems on website

RECALL

$f(x)$	$f'(x)$
3	0
$7x$	7
$11x^2$	$22x$
$-9x^6$	$-54x^5$
$3x^3 - 2x^2$	$9x^2 - 4x$
$6x$	6
8	0

23.4 Instantaneous Rate of Change

and 23.9 Higher Derivatives

Cont'd

RECALL Motion in a straight line

$s(t)$: displacement after t seconds (m)

$v(t)$: velocity " (m/s)

$a(t)$: acceleration " (m/s²)

FACT

$$v(t) = s'(t)$$

$$a(t) = v'(t)$$

Ex. $s(t) = t^2 + 5t$

s : in m, t : in s

a) Find the velocity at $t = 3$ s

$$v(t) = s'(t)$$

$$v(t) = 2t + 5$$

$$v(3) = 11 \text{ m/s}$$

b) Find the time when velocity = 13 m/s.

$$v(t) = 2t + 5$$

$$\text{Set } v(t) = 13 : \quad 13 = 2t + 5$$

$$8 = 2t$$

$$4 = t$$

$$t = 4 \text{ s}$$

c) Is acceleration constant?

$$a(t) = v'(t)$$

$$a(t) = 2 \text{ m/s}^2$$

Yes

Ex: Find all derivatives of $f(x) = x^4 + 7x^2$

$$f'(x) = 4x^3 + 14x$$

$$f''(x) = 12x^2 + 14$$

$$f'''(x) = 24x$$

$$f^{(4)}(x) = 24$$

$$f^{(5)}(x) = 0$$

$$f^{(6)}(x) = 0 \text{ etc.}$$

$$\left[\begin{array}{l} \text{FACT} \\ v(t) = s'(t) \\ a(t) = v'(t) \text{ or } a(t) = s''(t) \end{array} \right]$$

Ex: $s(t) = 8t^3 - 12t^2 + 5$

Find velocity and acceleration.

$$v(t) = 24t^2 - 24t$$

$$a(t) = 48t - 24$$

$$v(t) = A'(t) \quad \text{or} \quad v = \frac{ds}{dt}$$

"Velocity is the (instantaneous) rate of change of displacement with respect to time"

$$\begin{aligned} \text{UNITS :} \\ \text{OF VELOCITY} & \quad \frac{(\text{unit of } s)}{(\text{unit of } t)} \\ & = \frac{\text{m}}{\text{s}} \end{aligned}$$

Ex: A circular oil spill has radius r and area A . Find the (instantaneous) rate of change of area w.r.t. radius, with respect to



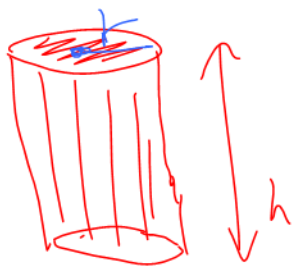
$$A = \pi r^2$$

$$A'(r) \quad \text{or} \quad \frac{dA}{dr} = 2\pi r$$

$$\text{UNITS FOR } \frac{dA}{dr} = \frac{\text{unit of } A}{\text{unit of } r} = \frac{\text{m}^2}{\text{m}}$$

Ex: A cylinder is heated and expands such that its height is always 3 times its radius.

Find the (instantaneous) rate of change of volume w.r.t. radius when radius = 10 cm.



$$V = \pi r^2 h$$

$$V = 3\pi r^3$$

$$\boxed{h = 3r}$$

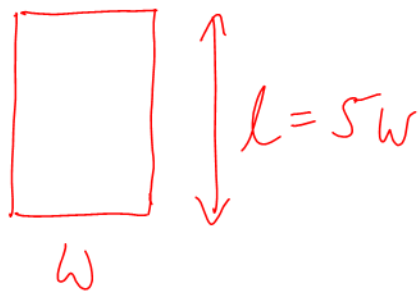
$$V'(r) \quad \text{or} \quad \frac{dV}{dr} = 9\pi r^2$$

$$V'(10) \quad \text{or} \quad \left. \frac{dV}{dr} \right|_{r=10} = 900\pi$$

$\frac{\text{cm}^3}{\text{cm}}$
 units of V
 units of r

Ex: A rectangular metal plate has length = 5(width) as it cools.

Find the (instantaneous) rate of change of the plate's area w.r.t. width.



$$A = lw$$

$$A = 5w^2$$

$$A'(w) \quad \text{or} \quad \frac{dA}{dw} = 10w \quad \frac{m^2}{m}$$

23.4 #19

Find the velocity if

$$s = 3t - \frac{6}{5t}$$

not a polynomial
Use the limit definition

$$v(t) = s'(t)$$

$$= \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[3(t+h) - \frac{6}{5(t+h)} - 3t + \frac{6}{5t} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[\cancel{3t} + 3h - \cancel{3t} + \frac{6}{5t} - \frac{6}{5(t+h)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[3h + \frac{6(t+h)}{5t(t+h)} - \frac{6t}{5t(t+h)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[3h + \frac{\cancel{6t} + 6h - \cancel{6t}}{5t(t+h)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}}{h} \left[3 + \frac{6}{5t(t+h)} \right]$$

$$= 3 + \frac{6}{5t^2}$$