

Section Order in Ch 23

23.5

23.4 and 23.9

23.6

23.7

23.8

23.5 Derivatives of Polynomials

$$\frac{d}{dx} [c] = 0$$

↑
any real #

"The derivative of a constant is 0"

Ex:

$f(x)$	$f'(x)$
7	0
$-4\sqrt{3}$	0
π^3	0
8	0
193	0

$$\frac{d}{dx} [x^n] = nx^{n-1}$$

$$n = 1, 2, 3, \dots$$

Ex:

$f(x)$	$f'(x)$
x	$1x^0 = 1$
x^2	$2x^1 = 2x$
x^3	$3x^2$
x^4	$4x^3$
x^{90}	$90x^{89}$

$$\frac{d}{dx} [c u] = c \frac{du}{dx}$$

any real # function of x

Ex:

$$\begin{aligned} & \frac{d}{dx} [2x^7] \\ &= 2(7x^6) \\ &= 14x^6 \end{aligned}$$

Ex:

$f(x)$	$f'(x)$
$3x^6$	$18x^5$
$-2x^4$	$-8x^3$
$5x^2$	$10x$

$f(x)$	$f'(x)$
$5x$	$5(1) = 5$
$7x$	7
$-3x$	-3

$$\frac{d}{dx} [u + v] = \frac{du}{dx} + \frac{dv}{dx}$$

$\uparrow \quad \uparrow$
 functions
 of x

Ex: $\frac{d}{dx} [x^{100} + x^{47}] = 100x^{99} + 47x^{46}$

Summarize

$f(x)$	$f'(x)$
4	0
$12x$	12
$13x^2$	$26x$

Ex: Differentiate

a) $f(x) = x^4 + 17\sqrt{3}$

$$f'(x) = 4x^3 + 0 = 4x^3$$

$$b) \quad f(x) = x^8 - 2x$$

$$f'(x) = 8x^7 - 2$$

$$c) \quad f(x) = 12x^6 - 9x + 11$$

$$f'(x) = 72x^5 - 9$$

$$d) \quad f(x) = 3x^4 - 7x^3 + 6x^2 + 7$$

$$f'(x) = 12x^3 - 21x^2 + 12x$$

Ex: Find the slope of the tangent line to $y = 2x^3 - 5x + 2$ at $x = -1$.

slope of the tangent line $y' = 6x^2 - 5$

$$y' \Big|_{x=-1} = 6(-1)^2 - 5$$

$$= 1$$

Ex: Where does $f(x) = x^3 - 3x$ have a horizontal tangent line?

———— A horizontal line has slope = 0.

Slope of the tangent line $f'(x) = 3x^2 - 3$

Horizontal tangent line: Set $3x^2 - 3 = 0$

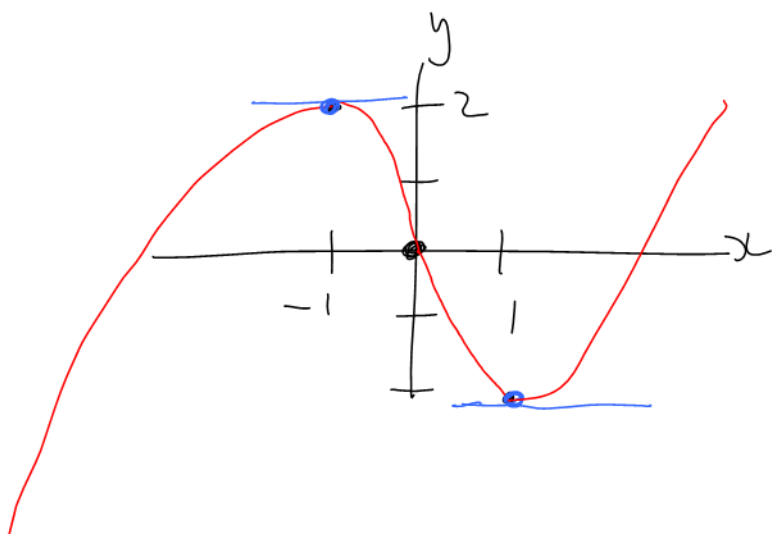
$$3(x^2 - 1) = 0$$

$$3(x-1)(x+1) = 0$$

$$x = \pm 1$$

Points $(x, y) = (1, -2)$ or $(-1, 2)$

plug into
 $f(x) = x^3 - 3x$



$$f(x) = x^3 - 3x$$

x	$f(x) = x^3 - 3x$
-1	2
0	0
1	-2

Ex: Find the derivative

a) $f = \pi r + 7r^{10}$

$$f'(r) \text{ or } \frac{df}{dr} = \pi + 70r^9$$

$$b) \quad V = 12m^3 - 8m^2$$

$$V' \text{ or } V'(m) \text{ or } \frac{dV}{dm} = 36m^2 - 16m$$

$$\underline{\text{Ex:}} \text{ Find } \frac{d}{dt} [t^6 + t^5 + t^4 \sqrt{r} + r t + r^5]$$

variable

(r is a constant)

$$= 6t^5 + 5t^4 + 4t^3(\sqrt{r}) + r + 0$$

$$= 6t^5 + 5t^4 + 4\sqrt{r}t^3 + r$$

Finding Derivatives

Use these rules unless the question says to use the "limit definition" of the derivative.

23.4 Instantaneous Rate of Change and 23.9 Higher Derivatives

Recall $f'(x)$ represents m_{tan}
and instantaneous rate of change of $f(x)$.



$$y = f(x)$$

Def

Suppose an object moves in a straight line (e.g. car in a driveway).

$s(t)$: displacement after t seconds (m)
 $v(t)$: velocity " (m/s)
 $a(t)$: acceleration " (m/s²)

FACT

$$v(t) = s'(t)$$

$$a(t) = v'(t)$$

Ex: $s(t) = t^2 + 5t$

s : in m, t : in s

a) Find velocity at $t = 1$ s

$$v(t) = s'(t)$$

$$v(t) = 2t + 5$$

$$v(1) = 7 \text{ m/s}$$

b) Find the time when
the velocity = 6 m/s

$$v(t) = 2t + 5$$

$$v(t) = 6 : \quad 6 = 2t + 5$$

$$1 = 2t$$

$$0.5 = t$$

$$t = 0.5 \text{ s}$$

c) Find acceleration at $t = 1 \text{ s}$

$$a(t) = v'(t)$$

$$a(t) = 2 \quad (\text{constant})$$

$$a(1) = 2 \text{ m/s}^2$$