HW Problems on D2L or www.leahhoward.com/191hw2026.pdf

List of Problems is on p.3

Full Solutions on website

Do Suggested HW for Sections 23.1, 23.2

Suggested HW is <u>not to be handed in</u>.

23.2 The Slope of a Tangent Line Cont'd

$$m_{tan} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

<u>Warmup</u>: Given $f(x) = x^2 - 3x$, find:

a) $f(1) = 1^2 - 3(1) = -2$

b) $f(8) = 40$

c) $f(c) = c^2 - 3c$

d) $f(x+h) = (x+h)^2 - 3(x+h)$

<u>Ex</u>: Find m_{tan} to $\overset{f(x)}{\cancel{y}} = 2x^2 - 6x + 3$

$$m_{tan} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2 - 6(x+h) + 3 - (2x^2-6x+3)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) \cancel{-6x} - 6h \cancel{+3} - 2x^2 \cancel{+6x} \cancel{-3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 6h \cancel{-2x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x+2h-6)}{\cancel{h}}$$

$$= \lim_{h\to 0} 4x + 2h - 6$$

$$= 4x - 6$$

23.3 The Derivative

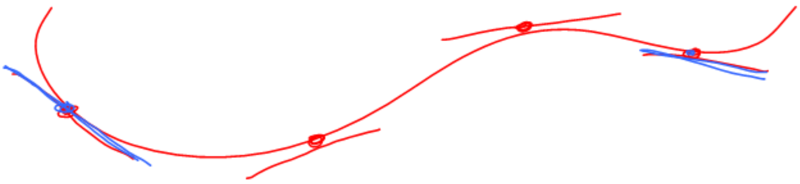
The derivative of $f(x)$ is written $f'(x)$

"f prime of x"

$f'(x)$ represents: slope of the tangent line

AND

instantaneous rate of change of $f(x)$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad (*)$$

Ex: $f(x) = 2x^3 - \pi$

Find $f'(x)$ and $f'(-1)$.

Hint: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^3 - \pi - (2x^3 - \pi)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) \cancel{-\pi} - 2x^3 \cancel{+\pi}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 \cancel{-2x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x^2 + 6xh + 2h^2)}{\cancel{h}}$$

$$= 6x^2 + 0 + 0$$

$$= 6x^2$$

$$f'(-1) = 6(-1)^2 = 6$$

Def

<u>Differentiate</u> $f(x)$: find $f'(x)$

Def

$f(x)$ is <u>differentiable</u> : $f'(x)$ is defined

From above,

$f'(x) = 6x^2$ is defined for all x

$f(x) = 2x^3 - \pi$ is differentiable for all x

Recap: Common Denominators

$$\frac{2}{x+h-1} - \frac{2}{x-1}$$

$$= \frac{2(x-1)}{(x+h-1)(x-1)} - \frac{2(x+h-1)}{(x-1)(x+h-1)}$$

$$= \frac{2(x-1) - 2(x+h-1)}{(x+h-1)(x-1)}$$

$$= \frac{\cancel{2x} - \cancel{2} - \cancel{2x} - 2h + \cancel{2}}{(x+h-1)(x-1)}$$

$\boxed{\text{Expand numerator. Keep denominator factored.}}$

$$= \frac{-2h}{(x+h-1)(x-1)}$$

Ex: $f(x) = x^2 + \frac{4}{x}$

Find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[(x+h)^2 + \frac{4}{x+h} - \left(x^2 + \frac{4}{x}\right)\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\cancel{x^2} + 2xh + h^2 + \frac{4}{x+h} - \cancel{x^2} - \frac{4}{x}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[2xh + h^2 + \frac{4x}{(x+h)x} - \frac{4(x+h)}{x(x+h)}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[2xh + h^2 + \frac{4x - 4(x+h)}{(x+h)x}\right]$$

$\vdots$

$$f'(x) = 2x - \frac{4}{x^2}$$