HW Problems on D2L
or www.leahhoward.com/191hw2026.pdf

List of problems is on p. 3

Full solutions on website

Do Suggested HW for Section 23.1

23.1 Limits

Review: Factoring

i) x^2+8x+7

$=(x+1)(x+7)$

Multiply to 7
Add to 8

ii) x^2-100

$=(x+10)(x-10)$

iii) $2x^2+5x+2$

ac Method
add to 5
multiply to 4

1,4

$= 2x^2 + 1x + 4x + 2$

$= x(2x+1) + 2(2x+1)$

$= (x+2)(2x+1)$

Ex: Find $\lim_{x \to 5} \frac{x^2-25}{x-5}$

Not continuous at $x=5$ (get $\frac{0}{0}$ when we sub $x=5$)

$= \lim_{x \to 5} \frac{(x-5)(x+5)}{x-5}$

Continuous at $x=5$

$= 5+5$

$= 10$

Ex: Find $\lim_{x \to -4} \frac{x^2-3x-28}{2x+8}$ $\left(\frac{0}{0}\right)$

$= \lim_{x \to -4} \frac{(x-7)(x+4)}{2(x+4)}$

$= \frac{-11}{2}$

Ex: Find $\lim_{x \to 0} \frac{x^2+9x}{x^2+x}$ $\left(\frac{0}{0}\right)$

$$= \lim_{x \to 0} \frac{x(x+9)}{x(x+1)}$$

$$= \frac{0+9}{0+1}$$

$$= 9$$

Ex: Find $\lim_{x \to 2} \frac{x^2+24}{x+11}$

$$= \frac{28}{13}$$

Ex: Find $\lim_{x \to 8} \frac{1}{x-8}$

Can't sub in ($\frac{1}{x-8}$ is undefined at $x=8$)

Can't factor.

Last resort: table of values.

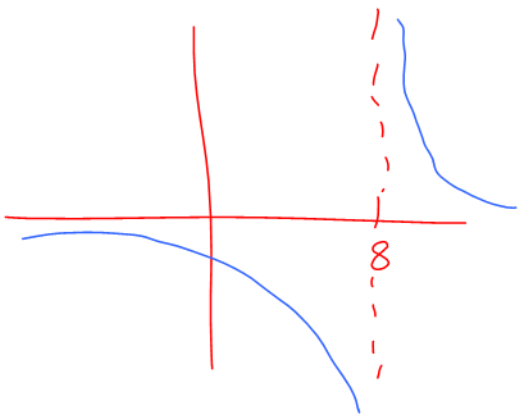
x	$\frac{1}{(x-8)}$
7.9	-10
7.99	-100
8.01	100
8.1	10

As $x \to 8$ from the left, $\frac{1}{x-8} \to -\infty$

As $x \to 8$ from the right, $\frac{1}{x-8} \to +\infty$

$\lim_{x \to 8} \frac{1}{x-8}$ is undefined

ASIDE

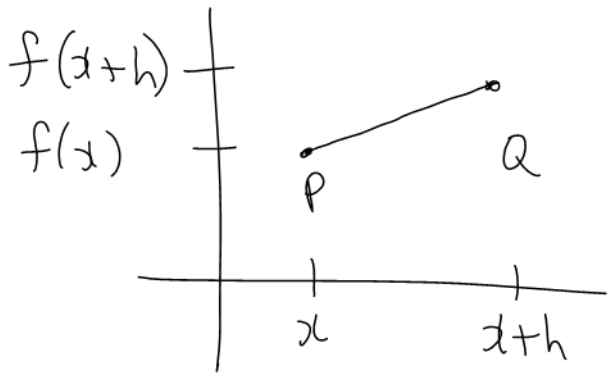


23.2 The Slope of a Tangent Line

Tangent line to a curve: a line that touches the curve at a single point.

Slope of the tangent line is written m_{tan}.

Consider a short line segment PQ.
Let h = small positive number (e.g. $h = 0.1$)



Slope of line segment PQ : $m_{PQ} = \frac{\text{rise}}{\text{run}}$

$$= \frac{f(x+h) - f(x)}{h}$$

As $h \to 0$, $m_{PQ} \to m_{tan}$.

FACT

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (*)$$

Ex: Find m_{tan} to $\overset{f(x)}{y} = x^2 + 5$ at the point $(x, y) = (1, 6)$.

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

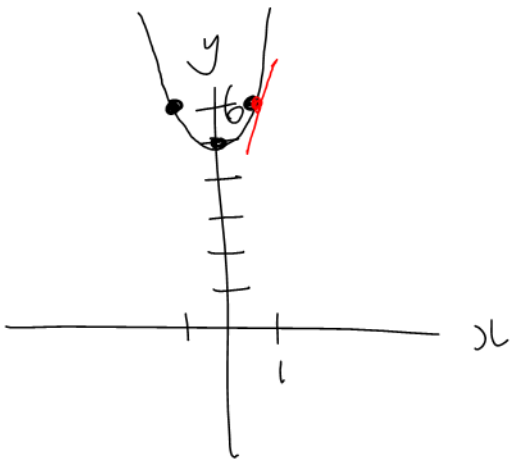
$$= \lim_{h \to 0} \frac{(1+h)^2 + 5 - 6}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 + 5 - 6}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2+h)}{h}$$

$$= 2$$



$y = x^2 + 5$

Ex: Find m_{tan} to $\cancel{y}\ f(x) = x^2 + 4x$

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - [x^2 + 4x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4x} + 4h - \cancel{x^2} - \cancel{4x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 4)}{\cancel{h}}$$

$$= 2x + 4$$

ASIDE:

When $x = 0$, $m_{tan} = 4$

$x = 2$, $m_{tan} = 8$

$x = -3$, $m_{tan} = -2$