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→ Math 191

Big ideas in Calculus:

1) Rates of change: velocity, acceleration
2) Areas, volumes, centres of mass etc.

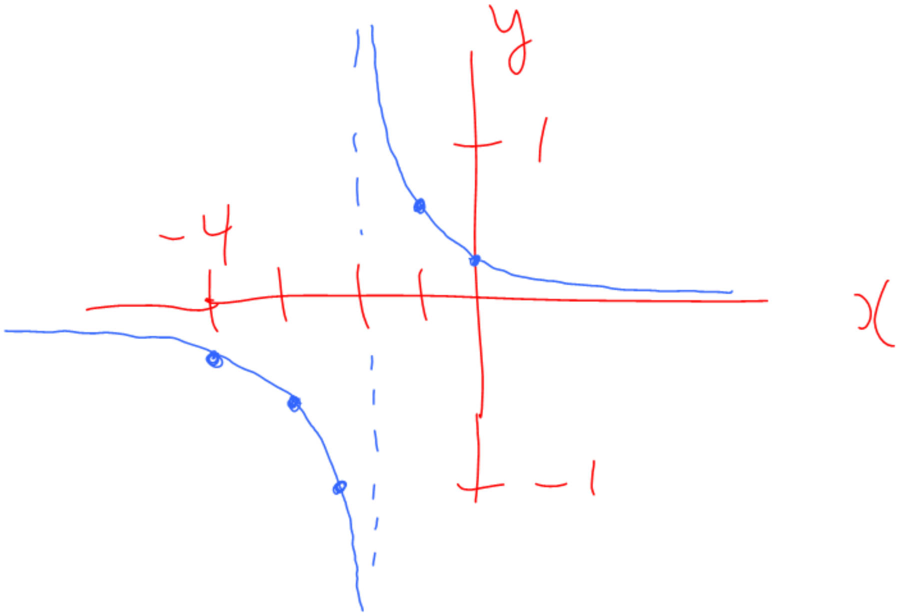
23.1 Limits

A function is <u>continuous</u> at an x-value if there is no jump or hole there.

<u>Ex</u>: $f(x) = \dfrac{1}{2x+4}$

Where is $f(x)$ continuous?

x	$f(x) = \frac{1}{2x+4}$
-4	$\frac{-1}{4}$
-3	$\frac{-1}{2}$
-2.5	-1
-2	undefined
-1	$\frac{1}{2}$
0	$\frac{1}{4}$



$f(x)$ is continuous for $x \neq -2$

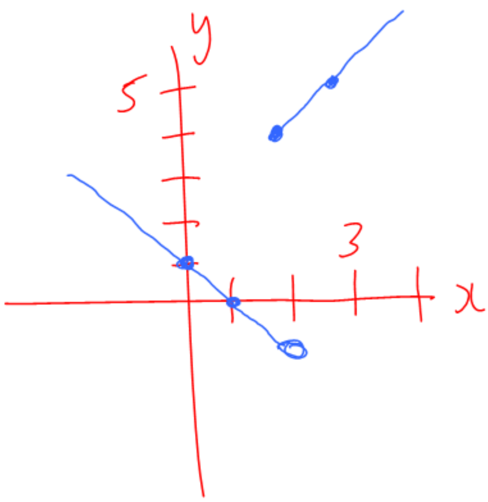
Alternatively:

$f(x)$ is continuous for all real numbers, except $x = -2$.

Ex.: $$f(x) = \begin{cases} x+2, & x \geq 2 \\ 1-x, & x < 2 \end{cases}$$

where is $f(x)$ continuous?

	x	$f(x)$	
$x<2$	0	1	$f(x)=1-x$
	1	0	
$x \geq 2$	2	4	$f(x)=x+2$
	3	5	

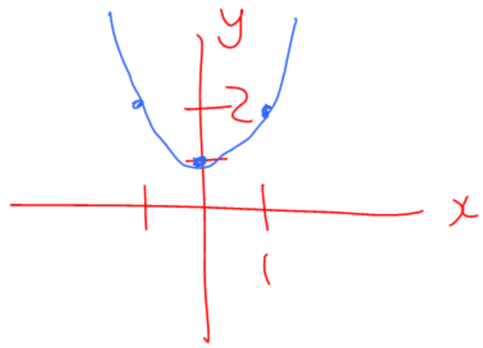


$f(x)$ is continuous for $x \neq 2$

Ex: $f(x) = x^2+1$

Where is $f(x)$ continuous?

x	$f(x)$
-1	2
0	1
1	2



$f(x)$ is continuous for all x

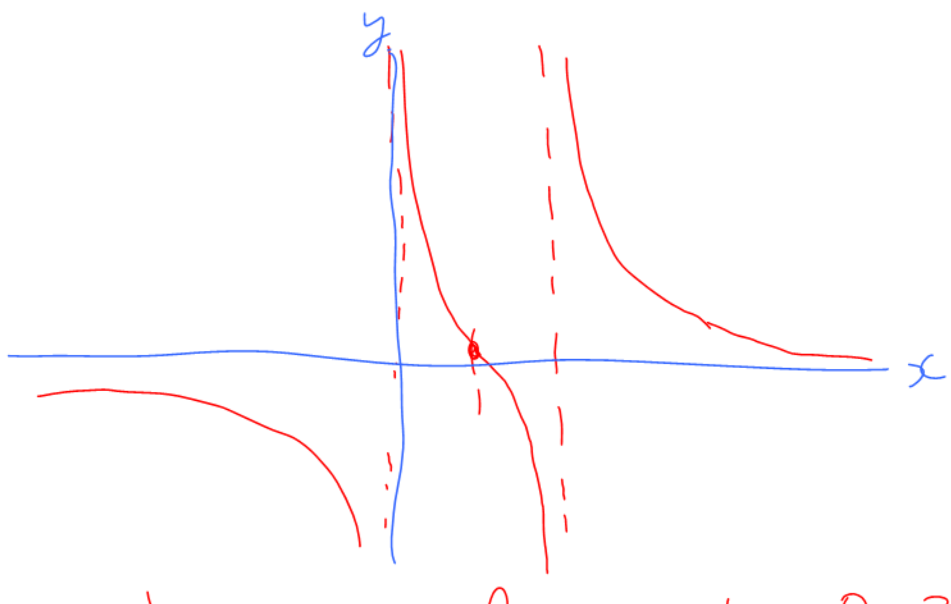
Ex: $f(x) = \dfrac{x-1}{x^2-2x}$

Where is $f(x)$ continuous?

Quick way to Graph:

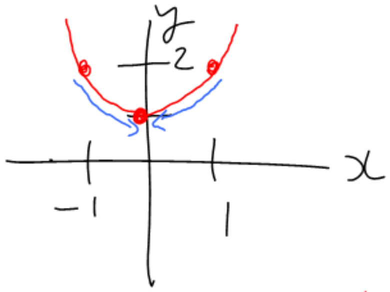
Google "Wolfram Alpha"

Type "Graph $y=(x-1)/(x\wedge2-2x)$"



$f(x)$ is continuous for $x \neq 0, 2$

Ex: $f(x) = x^2+1$



the y-value is approaching 1

As $x \to 0$ from the left, $f(x) \to 1$

As $x \to 0$ from the right, $f(x) \to 1$

We write $\lim_{x \to 0} f(x) = 1$

" The limit of $f(x)$ as x approaches 0 is 1 "

FACT

If $f(x)$ is continuous at $x=a$
then $\lim_{x \to a} f(x) = f(a)$

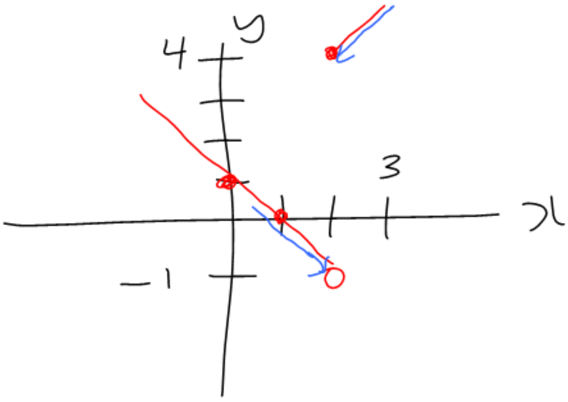
Ex: $\lim_{x \to 3} x^2+1$

$= 3^2+1$

$= 10$

because $f(x) = x^2+1$ is continuous at $x=3$

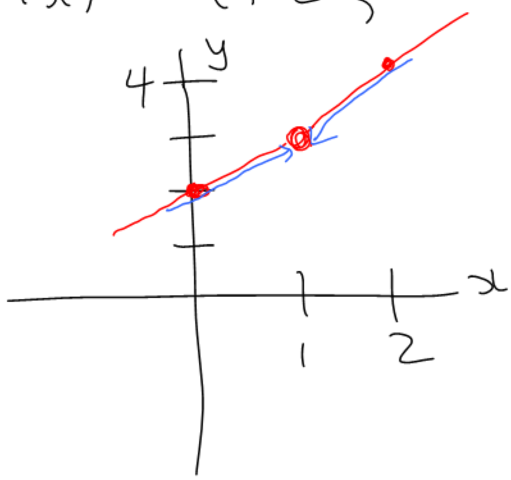
Ex: $f(x) = \begin{cases} x+2, & x \geq 2 \\ 1-x, & x < 2 \end{cases}$



As $x \to 2$ from the left, $f(x) \to -1$ } don't agree
As $x \to 2$ " right, $f(x) \to 4$ }

$\lim_{x \to 2} f(x)$ does not exist

Ex: $f(x) = x + 2, \quad x \neq 1$



x	$f(x) = x+2$
0	2
2	4

$$\lim_{x \to 1} f(x) = 3$$

The limit exists even though $f(x)$ is not defined at $x = 1$.

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