Math 252 Final Exam Review

1. Find an implicit solution. Don't simplify the solution.
$(y + y\cos xy)dx + (x + x\cos xy + \frac{2}{y})dy = 0$

2. Find an implicit solution. Don't simplify the solution.
$y' + \frac{y}{x} = x^3y^3$

3. Find an explicit solution.
$(x^2 + y^2)dx - xydy = 0$

4. A car has mass m. The car's mass times acceleration is numerically proportional to its negative velocity. The car's initial velocity is v_0. Set up and solve a DE to find the car's velocity as a function of time.

5. Solve using Variation of Parameters:
$x^2y'' - xy' + y = x^3$

6. Find the general solution given $y_1 = x$:
$x^3y'' - xy' + y = 0$

7. Solve using Undetermined Coefficients:
$y''' - y'' = 2 + 2\cos x$

8. A 1 kg mass is attached to a spring with spring constant 9 N/m. There is no damping. The mass is initially 2 m below equilibrium position, moving upwards at 1 m/s.
a) Find $x(t)$.
b) Write $x(t)$ as $A\sin(\omega t + \phi)$.

9. Find the first two nonzero terms of y_1 and y_2:
$(x + 2)y'' + y = 0$

10. Solve using the Laplace transform given $y(0) = 0, y'(0) = 1$:
$y'' + 4y = \begin{cases} \cos 2t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$

11. Use the Laplace transform to solve:
$y'' + 4y = 3e^{-t} + \cos 2t, y(0) = 1, y'(0) = 2$

12. Use the Laplace transform to solve:
$y'' + 4y' + 13y = 0, y(0) = 7, y'(0) = 3$

13. Solve $\vec{X}' = \begin{bmatrix} 2 & 4 \\ 6 & 4 \end{bmatrix} \vec{X}$.

14. Solve $\vec{X}' = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} \vec{X}$.

15. Solve $\vec{X}' = \begin{bmatrix} -1 & -5 \\ 5 & -1 \end{bmatrix} \vec{X}$.