① a) $1.75 \frac{\text{units}}{\text{month}} \cdot 12 \frac{\text{months}}{\text{year}}$

$= 21 \frac{\text{units}}{\text{year}}$

$\frac{dz}{dt} = 0.17z - 21$

b) $a = g - kv$

But $a = \frac{dv}{dt}$

$\frac{dv}{dt} = g - kv$

(2) $y'' + 8xy' + (4+16x^2)y = 0$ standard form

$P(x) = 8x$

$\int P(x)dx = 4x^2$

$-\int P(x)dx = -4x^2$

$e^{-\int P(x)dx} = e^{-4x^2}$

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

$$= e^{-2x^2} \int \frac{e^{-4x^2}}{(e^{-2x^2})^2} dx$$

$$= e^{-2x^2} \int \frac{e^{-4x^2}}{e^{-4x^2}} dx$$

$$= e^{-2x^2} \int 1 \, dx$$

$$= e^{-2x^2} (x)$$

$$= x e^{-2x^2}$$

(3) $y'' + 6y' + 13y = 0$

$m^2 + 6m + 13 = 0$

$m = \frac{-6 \pm \sqrt{36 - 52}}{2} = \frac{-6 \pm \sqrt{-16}}{2} = -3 \pm 2i$ ($\sqrt{-16} = 4i$)

$$\boxed{y = e^{-3x}(C_1 \cos 2x + C_2 \sin 2x)}$$

$y = 3e^{-3\pi}$, $x = \pi$: $3e^{-3\pi} = e^{-3\pi}(C_1) \Rightarrow C_1 = 3$

$$\boxed{y' = e^{-3x}(-2C_1 \sin 2x + 2C_2 \cos 2x) - 3e^{-3x}(C_1 \cos 2x + C_2 \sin 2x)}$$

$y' = -19e^{-3\pi}$, $x = \pi$: $-19e^{-3\pi} = e^{-3\pi}(2C_2) - 3e^{-3\pi}(C_1)$

$-19 = 2C_2 - 3C_1$

$C_1 = 3 \rightarrow \quad -19 = 2C_2 - 3C_1$

$-19 = 2C_2 - 9$

$C_2 = -5$

$$\boxed{y = e^{-3x}(3 \cos 2x - 5 \sin 2x)}$$

(4) a) $m^2+2m-35=0$

$(m+7)(m-5)=0$

$m=-7, 5 \Rightarrow y_1=e^{-7x}, y_2=e^{5x}$

$y_c = C_1 e^{-7x} + C_2 e^{5x}$

$y_p = A\cos 3x + B\sin 3x + C\cos 4x + D\sin 4x$

(not bad case)

b) $m^3-2m^2=0$

$m^2(m-2)=0$

$m=0,0,2$

$y_1=e^{0x}=1, \quad y_2=1(x)=x, \quad y_3=e^{2x}$

$y_c = C_1 + C_2 x + C_3 e^{2x}$

$y_p = \cancel{Ax} + \cancel{B} + C\cos 2x + D\sin 2x$ (bad case)

Ax^3+Bx^2

$y_p = Ax^3 + Bx^2 + C\cos 2x + D\sin 2x$

(5) $m^2+12m+36=0$

$(m+6)^2=0$

$m=-6,-6 \Rightarrow y_1=e^{-6x},\ y_2=xe^{-6x}$

$y_c=(C_1+C_2x)e^{-6x}$

$y_p=Ae^{-2x}+B\cos 3x+C\sin 3x+D$ (not bad cam)

Sub into DE

$$\begin{cases} y_p=Ae^{-2x}+B\cos 3x+C\sin 3x+D \\ y_p'=-2Ae^{-2x}-3B\sin 3x+3C\cos 3x \\ y_p''=4Ae^{-2x}-9B\cos 3x-9C\sin 3x \end{cases}$$

$y''+12y'+36y=11e^{-2x}+5\cos 3x+14$

$4Ae^{-2x}-9B\cos 3x-9C\sin 3x$

$+12(-2Ae^{-2x}-3B\sin 3x+3C\cos 3x)$

$+36(Ae^{-2x}+B\cos 3x+C\sin 3x+D)$

$=11e^{-2x}+5\cos 3x+0\sin 3x+14$

$\overbrace{[4A-24A+36A]}^{16A}e^{-2x}+\overbrace{[-9B+36C+36B]}^{27B+36C}\cos 3x$

$+\underbrace{[-9C-36B+36C]}_{27C-36B}\sin 3x+36D=11e^{-2x}+5\cos 3x+0\sin 3x+14$

$16A=11 \Rightarrow A=\frac{11}{16}$ $\quad 36D=14 \Rightarrow D=\frac{14}{36}=\frac{7}{18}$ $\rightarrow$

$$27B + 36C = 5 \quad (1)$$
$$-36B + 27C = 0 \quad (2)$$

$$(2) \Rightarrow \quad C = \frac{36B}{27} = \frac{4}{3}B$$

$$\frac{4}{3}B \to (1): \quad 27B + 48B = 5$$
$$75B = 5$$
$$B = \frac{1}{15}$$

$$C = \frac{4}{3}B = \frac{4}{45}$$

$$y = y_c + y_p$$
$$y = (C_1 + C_2 x)e^{-6x} + \frac{11}{16}e^{-2x} + \frac{1}{15}\cos 3x + \frac{4}{45}\sin 3x + \frac{7}{18}$$