1. [4 marks] Find y':

a) $y = \frac{1}{3}\csc 6x^2$

$y' = \frac{1}{3}[-\csc 6x^2 \cot 6x^2](12x)$

$= -4x \csc 6x^2 \cot 6x^2$

or $-4x \csc(6x^2)\cot(6x^2)$

b) $y = 3\cos^4 \frac{x}{6}$

$y = 3[\cos \frac{x}{6}]^4$

$y' = 12[\cos \frac{x}{6}]^3 (-\sin \frac{x}{6})(\frac{1}{6})$

$= -2\cos^3 \frac{x}{6} \sin \frac{x}{6}$

2. [2 marks] We want to approximate a solution to $3x^4 - 4x^3 - 5 = 0$.
Use $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ with $x_0 = -1$ to find x_1.
Round x_1 to two decimal places. You do **not** need to find x_2.

$f(x) = 3x^4 - 4x^3 - 5$

$f'(x) = 12x^3 - 12x^2$

x_n	$f(x_n)$ $= 3x^4 - 4x^3 - 5$	$f'(x_n)$ $= 12x^3 - 12x^2$	x_{n+1}
-1	2	-24	-0.9167

$x_1 \approx -0.92$

3. [4 marks] An object's position (in m) after t seconds is given below. Find the magnitude and direction of the object's velocity at $t = 2$ seconds. Round your values to one decimal place.

$x = 1 - 3t^3, \quad y = 1 + 2t^2$

$v_x = -9t^2, \quad v_y = 4t$

At $t=2$: $v_x = -36, \quad v_y = 8$

Speed $v = \sqrt{(v_x)^2 + (v_y)^2}$

$= \sqrt{(-36)^2 + 8^2}$

≈ 36.9 m/s

direction $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$ (+180°?)

$= \tan^{-1}\left(\frac{8}{-36}\right) + 180°$

$\approx 167.5°$

Add 180° when $v_x < 0$

4. [4 marks] When a spherical balloon is inflated, its volume increases by 360π cm^3/s. At what rate is the radius increasing when the radius is 3 cm? Hint: $V = \frac{4}{3}\pi r^3$.

$V = \frac{4}{3}\pi r^3$

Take $\frac{d}{dt}$:

$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

Sub

$\frac{dV}{dt} = 360\pi$, $r = 3$: $360\pi = 4\pi(9)\frac{dr}{dt}$

$\frac{dr}{dt} = 10 \ \frac{cm}{s}$

5. [4 marks] Let $f(x) = 15x^3 - x^5$. For which x-values is $f(x)$ increasing?

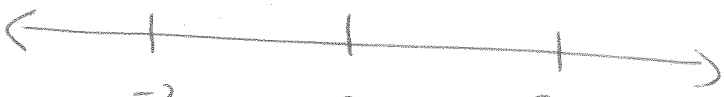
$f'(x) = 45x^2 - 5x^4$

Set $f'(x) = 0$: $0 = 45x^2 - 5x^4$

$0 = 5x^2(9 - x^2)$

$0 = 5x^2(3 - x)(3 + x)$

$x = 0, 3, -3$



$f'(x)$	$\ominus$	...	$\oplus$	0	$\oplus$	$\ominus$
$f(x)$	DEC		INC		INC	DEC

$f(x)$ is increasing for $-3 < x < 0$ or $0 < x < 3$.

ALSO ACCEPTABLE:

$f(x)$ is increasing for $-3 < x < 3$

6. [3 marks] Let $f = 6x^4$. Find $\frac{df}{f}$ given that $\frac{dx}{x} = 0.03$.

$$\frac{df}{dx} = 24x^3$$

$$df = 24x^3\,dx$$

$$\frac{df}{f} = \frac{24x^3\,dx}{6x^4}$$

$$= \frac{4\,dx}{x}$$

$$= 4\left(\frac{dx}{x}\right)$$

$$= 4(0.03)$$

$$= 0.12$$

7. [4 marks] Find the value of r that minimizes $f = 2\pi r^2 + 2\pi rh$, given that $r^2h = 16$. You do **not** need to verify that your answer is a minimum.

$h = \frac{16}{r^2} \rightarrow f = 2\pi r^2 + 2\pi r h$

$f = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$f = 2\pi r^2 + 32\pi r^{-1}$

$f' = 4\pi r - 32\pi r^{-2}$

Set $f' = 0$: $0 = 4\pi r - 32\pi r^{-2}$

Multiply by r^2: $0 = 4\pi r^3 - 32\pi$

$0 = 4\pi(r^3 - 8)$

$r^3 - 8 = 0$

$r^3 = 8$

$\sqrt[3]{r^3} = \sqrt[3]{8}$

$r = 2$