(1) $a_n = a_m + (n-m)d$

$n=37$
$m=21$: $a_{37} = a_{21} + 16d$

$a_{37} = -239$
$a_{21} = 33$: $-239 = 33 + 16d$

$-272 = 16d$

$-17 = d$

$\boxed{d = -17}$

(2) $a_n = a_m + (n-m)d$

$m=3$: $a_n = a_3 + (n-3)d$

$a_3 = 26$
$d = 4$
$a_n = 298$: $298 = 26 + (n-3)(4)$

$272 = (n-3)(4)$

$68 = n-3$

$71 = n$

$\boxed{a_{71} = 298}$

(3) Geometric Sequence

$r = \frac{4}{5}$

$a_1 = 11$

$r = \frac{\left(\frac{44}{5}\right)}{11} = \frac{44}{5} \times \frac{1}{11} = \frac{4}{5}$

$a_n = a_m r^{n-m}, \quad n \geq m$

$m=1$: $a_n = a_1 r^{n-1}, \quad n \geq 1$

$a_n = 11\left(\frac{4}{5}\right)^{n-1}, \quad n \geq 1$

(4) Geometric Series

$a_1 = 7$

$r = \frac{1}{3}$

$r = \frac{\left(\frac{7}{3}\right)}{7} = \frac{7}{3} \times \frac{1}{7} = \frac{1}{3}$

$-1 < r < 1$ ✓

$S_\infty = \frac{a_m}{1-r}$

$m=1$: $S_\infty = \frac{a_1}{1-r}$

$a_1 = 7$, $r = \frac{1}{3}$: $S_\infty = \left(\frac{7}{1-\frac{1}{3}}\right)$

$= 10.5$ or $\frac{21}{2}$

(5) $\sum_{n=3}^{17} (6n+1)$

$= 19 + 25 + 31 + \ldots + 103$

Arithmetic Series (with $d=6$)

$a_3 = 19$, $a_{17} = 103$

k = # of terms $= 17 - 3 + 1 = 15$

$S_k = \frac{k}{2}(a_m + a_n)$

$k=15$
$m=3$: $S_{15} = \frac{15}{2}(a_3 + a_{17})$
$n=17$

$a_3 = 19$: $S_{15} = \frac{15}{2}(19 + 103)$
$a_{17} = 103$

$S_{15} = 915$

ALTERNATIVELY:

Use $S_k = \frac{k}{2}[2a_m + (n-m)d]$

$m=3$
$n=17$: $S_{15} = \frac{15}{2}[2a_3 + 14d]$
$k=15$

$a_3 = 19$: $S_{15} = \frac{15}{2}[2(19) + 14(6)]$
$d=6$

$S_{15} = 915$

(6) a) $O(2^n)$

b) $O(\log n)$

c) $O(n)$

d) $6n(n+5) = 6n^2 + 30n$
this is $O(n^2)$.

(7) $\underbrace{7n^2 + 3(2^n)}_{O(2^n)}$, $\underbrace{6n(n+5)}_{O(n^2)}$, $\underbrace{2n + 3\log n}_{O(n)}$, $\underbrace{1000 + 3\log n}_{O(\log n)}$

(8) a) Continuous

b) Discrete

c) Qualitative