1. [2 marks] A feasible set has vertices $(5, 4), (4, 5)$ and $(2, 6)$.

a) Find the maximum value of $19x + 23y$ on the feasible set.

Point	$19x+23y$
$(5,4)$	187
$(4,5)$	191 $\Leftarrow$
$(2,6)$	176

The maximum value is 191.

b) State the point where that maximum value occurs.

It occurs at $(4,5)$

2. [4 marks] Find the equation of the line that passes through the point $(-3, 7)$ and is perpendicular to $y = -\frac{4}{5}x + 4$.

$y = -\frac{4}{5}x + 4$ has slope $= -\frac{4}{5}$

The perpendicular line has slope $= \frac{5}{4}$

$$y = mx + b$$

$$y = \frac{5}{4}x + b$$

Sub $x=-3, y=7$: $7 = \frac{-15}{4} + b$

$$\frac{28}{4} + \frac{15}{4} = b$$

$$\frac{43}{4} = b$$

$$y = \frac{5}{4}x + \frac{43}{4}$$

3. [4 marks] A company makes surfboards and paddleboards. Each surfboard takes 4 hours to manufacture and 2 hours to dye and generates \$115 of profit. Each paddleboard takes 4 hours to manufacture and 3 hours to dye and generates \$210 of profit. Each day the company has 48 manufacturing hours and 30 dyeing hours available. Let x be the number of surfboards made each day. Let y be the number of paddleboards made each day.

a) Write down the function that represents daily profit.

	Surfboards (x)	Paddleboards (y)	Available
Manufacture (hours)	4	4	48
Dye (hours)	2	3	30
Profit (\$)	115	210	//////////

Daily Profit $= 115x + 210y$

b) List all the inequalities that apply.

$4x + 4y \leq 48$ maximum available

$2x + 3y \leq 30$

$x \geq 0$

$y \geq 0$

4. [4 marks] Find the point where the lines $6x+3y=-12$ and $-4x+8y=88$ intersect.

$$6x+3y=-12$$
$$3y=-6x-12$$
$$y=-2x-4$$

$$-4x+8y=88$$
$$8y=4x+88$$
$$y=\frac{x}{2}+11$$

$$y=y$$
$$-2x-4=\frac{x}{2}+11$$

Multiply by 2:
$$-4x-8=x+22$$
$$-5x=30$$
$$x=-6$$

$x=-6 \rightarrow$ either line : $y=8$

$$\boxed{(-6,8)}$$

5. [3 marks] Find the equation of the line that passes through the points $(-10,9)$ and $(2,27)$.

$$m=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{(27-9)}{(2+10)}$$
$$=\frac{18}{12}$$
$$=\frac{3}{2}$$

$$y=mx+b$$
$$y=\frac{3}{2}x+b$$

Sub $x=2$, $y=27$: $27=3+b$
$$24=b$$

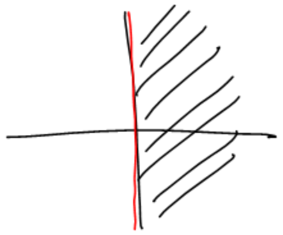
$$\boxed{y=\frac{3}{2}x+24}$$

6. [3 marks] Graph the feasible set. Shade in the region that satisfies all the inequalities.

$$\begin{aligned} x &\geq 0 \\ 2x + y &\leq 6 \\ y &\geq 4x \end{aligned}$$

$x = 0$

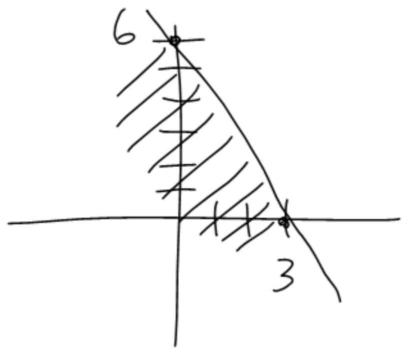
(vertical line)



Test $(1,0)$: YES

$2x + y = 6$

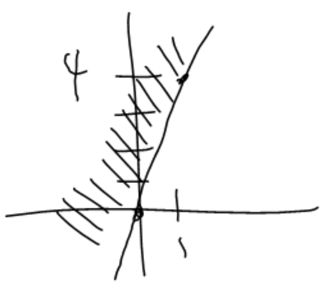
2 points: $(0,6)$, $(3,0)$



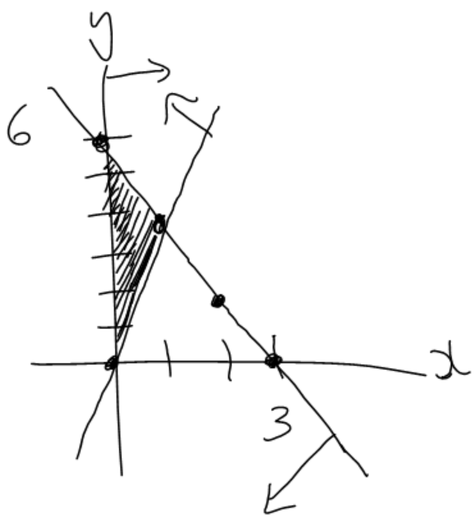
Test $(0,0)$: YES

$y = 4x$

2 points: $(0,0)$, $(1,4)$



Test $(1,0)$: No



7. [5 marks] Solve the following system using Gauss-Jordan Elimination:

$$\begin{aligned} x - 2y + 7z &= -2 \\ 3x - 4y + 17z &= 2 \\ 5x - 7y + 29z &= 2 \end{aligned}$$

$$\begin{array}{cccc} x & y & z & \# \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 7 & -2 \\ 3 & -4 & 17 & 2 \\ 5 & -7 & 29 & 2 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - 3R_1 \\ R_3 - 5R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 7 & -2 \\ 0 & 2 & -4 & 8 \\ 0 & 3 & -6 & 12 \end{array}\right]$$

$$\frac{R_2}{2} \left[\begin{array}{ccc|c} 1 & -2 & 7 & -2 \\ 0 & 1 & -2 & 4 \\ 0 & 3 & -6 & 12 \end{array}\right]$$

$$\begin{array}{l} R_1 + 2R_2 \\ \\ R_3 - 3R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 6 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x + 3z = 6 \implies \qquad y - 2z = 4 \implies$$

$$\boxed{\begin{aligned} x &= 6 - 3z \\ y &= 4 + 2z \\ z &= \text{any value} \end{aligned}}$$