

9.3 If-Then Statements; If and Only If Statements

Recall that the statement “if p then q ” is written: $p \rightarrow q$.

We can think of $p \rightarrow q$ as a promise.

Example: Consider the statement “If the client likes you then you get a promotion.”
Let’s fill in the table below.

Client Likes You	You Get a Promotion	Promise is Kept
T	T	T
T	F	(F) ← promise is broken
F	T	T
F	F	T

Fact: The statement $p \rightarrow q$ is false exactly when p is true and q is false.

Example: Build the truth table for $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: Build the truth table for $r \rightarrow (p \wedge q)$.

3 statements: p, q, r
Need $2 \times 2 \times 2 = 8$ rows

p	q	r	$p \wedge q$	$r \rightarrow (p \wedge q)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T
F	F	T	F	F
F	F	F	F	T

Notation: The statement “ p if and only if q ” is written: $p \leftrightarrow q$.

Fact: The statement “ p if and only if q ” is true exactly when p and q have the same truth values.

Example: Build the truth table for $p \leftrightarrow q$.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example: Build the truth table for $(p \vee q) \leftrightarrow r$.

3 statements: p, q, r
Need 8 rows

p	q	r	$p \vee q$	$(p \vee q) \leftrightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	T

Fact: Order of Operations

In a compound statement we apply \sim first, then \wedge , then \vee , then \rightarrow , then \leftrightarrow , unless brackets indicate otherwise.

Example: Bracket the statements below:

a) $q \wedge r \rightarrow p$

$(q \wedge r) \rightarrow p$

b) $\sim q \rightarrow r$

$(\sim q) \rightarrow r$

c) $p \leftrightarrow q \vee r$

$p \leftrightarrow (q \vee r)$

Comment: There is no consensus on where \oplus (exclusive or) fits into the order of operations, so statements involving \oplus should always be bracketed.