## 9.2 Truth Tables

In this section we'll look at how the truth values of $\sim p$, $p \vee q$, $p \wedge q$ depend on the truth values of $p$ and $q$.

**Fact:** $\sim p$ is true exactly when $p$ is false.

Let's build what is called the **truth table** for $\sim p$:

| $p$ | $\sim p$ |
|---|---|
| T | F |
| F | T |

(2 rows)

**Fact:** $p \vee q$ is true exactly when at least one of $p$ or $q$ is true.

**Example:** Build the truth table for $p \vee q$.

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

(4 rows)

**Fact:** $p \wedge q$ is true exactly when both $p$ and $q$ are true.

**Example:** Build the truth table for $p \wedge q$.

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

**Definition:** $p \oplus q$ means: p or q, but not both.
It is pronounced "p exclusive or q" or "p xor q".

**Fact:** $p \oplus q$ is true exactly when $p$ and $q$ have different truth values.

**Example:** Build the truth table for $p \oplus q$.

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

Inclusive Or: Do you take cream or sugar?
Exclusive Or: Should I go to class or not?

**Example:** Build the truth table for $\sim(p \wedge q)$.

| $p$ | $q$ | $p \wedge q$ | $\sim(p \wedge q)$ |
|---|---|---|---|
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |

**Example:** Build the truth table for $(p \oplus q) \vee (p \wedge q)$.

| $p$ | $q$ | $p \oplus q$ | $p \wedge q$ | $(p \oplus q) \vee (p \wedge q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | F | T |
| F | T | T | F | T |
| F | F | F | F | F |

**Example:** Build the truth table for $(p \wedge q) \vee r$.

The compound statement involves 3 statements: $p, q, r$.

Need $2 \times 2 \times 2 = 8$ rows

| $p$ | $q$ | $r$ | $p \wedge q$ | $(p \wedge q) \vee r$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | F | T |
| T | F | F | F | F |
| F | T | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |
| F | F | F | F | F |

**Example:** Build the truth table for $\boxed{(p \vee q) \oplus ((p \vee r) \wedge \sim p)}$.

The compound statement involves 3 statements: $p, q, r$.
Need 8 rows.

| $p$ | $q$ | $r$ | $p \vee q$ | $p \vee r$ | $\sim p$ | $(p \vee r) \wedge \sim p$ | |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | F | F | T |
| T | T | F | T | T | F | F | T |
| T | F | T | T | T | F | F | T |
| T | F | F | T | T | F | F | T |
| F | T | T | T | T | T | T | F |
| F | T | F | T | F | T | F | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | F | T | F | F |

**Definition:** A statement that is always true is called a **tautology**. A statement that is always false is called a **contradiction**.

**Example:** Is the following statement a tautology, a contradiction, or neither?
$\sim (p \vee q) \wedge q$

| $p$ | $q$ | $p \vee q$ | $\sim(p \vee q)$ | $\sim(p \vee q) \wedge q$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | T | F | F |
| F | T | T | F | F |
| F | F | F | T | F |

always false

$\sim(p \vee q) \wedge q$ is a contradiction.