9.1 Intro to Logic

We have two main goals in this chapter:
1) Break down compound statements into simple statements.
2) Decide whether a compound statement is true or false.

These skills are important when reading legal documents such as employment contracts or tenancy agreements.

Definition: A **simple statement** is a sentence that is either true or false.

Example: Here are some simple statements. State the truth value for each one.

a) Water freezes at 23°C.

F (false)

b) -2 is less than 3.

T (true)

Example: The following are **not** simple statements:

a) Tomatoes are delicious.

opinion

b) $2x + 1 \geq 7$

depends on x

c) Is it raining?

question

d) Be quiet!

command

Definition: Let p and q be simple statements. Here are four commonly-used **compound statements**. Let's write the notation for each compound statement.

a) p and q

$p \wedge q$

b) p or q

$p \vee q$

c) not p

$\sim p$

d) if p then q

$p \rightarrow q$

Comment: The compound statement "p or q" means: p or q or both.

Comment: The compound statement "if p then q" can be rephrased as "p implies q".

Example: Let p be: Paula is going to the party.
Let q be: Quinn is going to the party.
Write the following statements symbolically:

a) Quinn is not going to the party.

$\sim q$

b) It is not the case that Paula is not going to the party.

$\sim(\sim p)$ Alternatively: p

c) Quinn and Paula are both going to the party.

$q \wedge p$ Alternatively: $p \wedge q$

d) Quinn or Paula is going to the party.

$q \vee p$ Alternatively: $p \vee q$

Rephrased: At least one of them is going.

e) Paula is going to the party ~~but~~ and Quinn is not.

$p \wedge \sim q$

f) Paula and Quinn are not both going to the party.

$\sim(p \wedge q)$ Alternatively: $\sim p \vee \sim q$

Rephrased: At least one of them is not going.

g) Neither Paula nor Quinn is going to the party.

$\sim p \wedge \sim q$ Alternatively: $\sim (p \vee q)$

h) Paula or Quinn is going to the party ~~but~~ and not both.

$(p \vee q) \wedge \sim (p \wedge q)$

i) If Paula is going to the party then Quinn is going.

$p \to q$

j) If Quinn is going to the party then Paula is not going.

$q \to \sim p$

Definition: Two compound statements are **logically equivalent** if they have the same meaning.

Fact: Recall De Morgan's Laws from Section 3.2:
$(S \cup T)' = S' \cap T'$
$(S \cap T)' = S' \cup T'$
Here is the logic version of De Morgan's Laws:
$\sim (p \vee q)$ is logically equivalent to $\sim p \wedge \sim q$
$\sim (p \wedge q)$ is logically equivalent to $\sim p \vee \sim q$

Fact: Order of Operations
In a compound statement we apply $\sim$ first, then $\wedge$, then $\vee$, then $\to$,
unless brackets indicate otherwise.

Example: Bracket the expressions below:

a) $\sim p \wedge q$

$(\sim p) \wedge q$

b) $p \vee q \wedge r$

$p \vee (q \wedge r)$

c) $q \vee \sim p \to r$

$(q \vee (\sim p)) \to r$