5.2 Binomial Experiments

Definition: A **Bernoulli trial** is an experiment with only two outcomes, usually called "success" and "failure."

Notation: Let p be the probability of success on a Bernoulli trial.
Let q be the probability of failure on a Bernoulli trial.

Fact: For any Bernoulli trial, $q = 1 - p$. This is true because $p + q = 1$.

Example: We roll a fair die. Suppose a success is rolling a 4. Calculate p and q.

$p = P(\text{roll a } 4) = \frac{1}{6}$

$q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

P(roll is not a 4)

Definition: A **binomial experiment** is a sequence of n independent Bernoulli trials.

one trial does not affect the others

Example: We roll a fair die seven times. Let a success be rolling less than 3.
Calculate n, p and q.

binomial experiment

$n = \#$ of trials $= 7$

$p = P(\text{success on one trial})$
$= P(\text{a roll is less than } 3)$
$= \frac{2}{6}$
$= \frac{1}{3}$

$q = 1 - p = \frac{2}{3}$ ← probability of failure on one trial

Fact: The probability of exactly x successes in a binomial experiment is:

$(nCx)p^x q^{n-x}$

where n is the number of trials,
p is the probability of success on one trial, and
q is the probability of failure on one trial.

$P(x \text{ successes}) = (nCx) \quad p^{x} \quad q^{n-x}$

choose x of n trials to be successes

AND

probability of x successes in a row

AND

probability of $n-x$ failures in a row

Example: We roll a fair die five times. Find:

a) the probability of rolling exactly three 2's.

binomial experiment

$n = \#$ of trials $= 5$

$p = P(\text{roll a } 2) = \frac{1}{6}$

$q = 1 - p = \frac{5}{6}$

$x = \#$ of successes $= \#$ of 2's rolled

$$P(x=3) = (nCx)\, p^x q^{n-x}$$
$$= (5C3)\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$$
$$\approx 0.03$$

b) the probability of rolling more than three 2's.

$$P(x>3) = P(x=4) + P(x=5)$$
$$= (5C4)\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + (5C5)\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^0$$
$$\approx 0.003$$

c) the probability of rolling fewer than four 2's.

$$P(x<4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$
$$= 1 - [P(x=4) + P(x=5)]$$
$$\approx 1 - 0.003$$
$$\approx 0.997$$

Example: A basketball player makes 65% of his free throws. He takes three shots and he does not improve with practice.

a) Let X be the number of successful free throws the player makes. Find the probability distribution of X.

binomial experiment

$n = \#$ of trials $= 3$

$p = P(\text{a free throw is successful}) = 0.65$

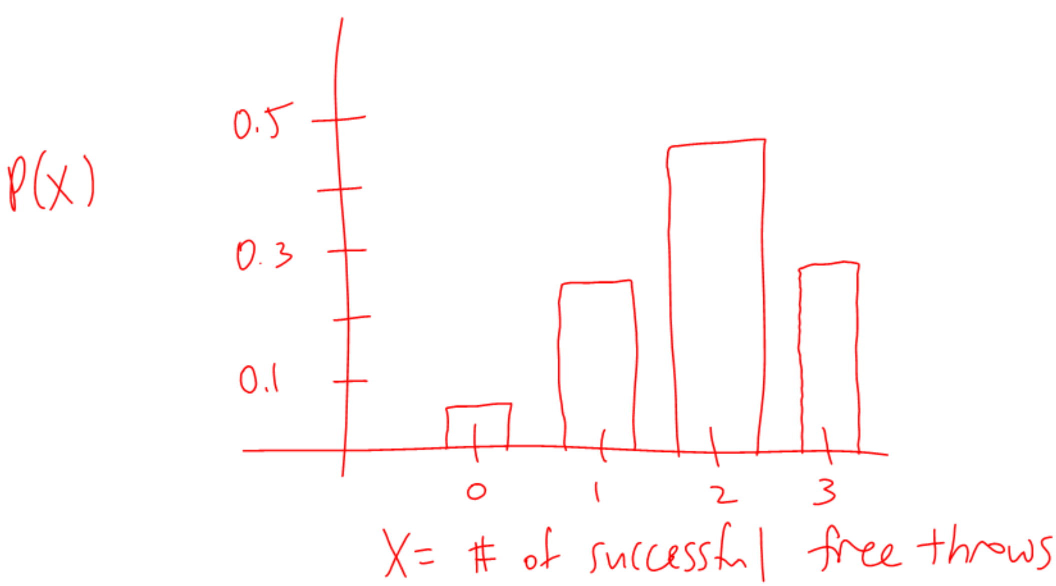
$q = 1 - p = 0.35$

$x = \#$ of successful free throws

X	$P(X)$
0	$(3C0)(0.65)^0(0.35)^3 \approx 0.04$
1	$(3C1)(0.65)^1(0.35)^2 \approx 0.24$
2	$(3C2)(0.65)^2(0.35)^1 \approx 0.44$
3	$(3C3)(0.65)^3(0.35)^0 \approx 0.27$

$(nCx)\, p^x q^{n-x}$

b) Draw a histogram (bar chart).



Recall from Section 5.1:

- The expected value of a random variable X is the theoretical average of X if the experiment were repeated infinitely-many times. It is sometimes called the **mean of** X.

- The expected value of X is written μ or $E(X)$. Feel free to use either notation.

- $\mu = x_1p_1 + x_2p_2 + \ldots + x_np_n$ where:

  $x_1, x_2, \ldots, x_n$ are the values of X

  $p_1, p_2, \ldots, p_n$ are their respective probabilities

Fact: Let X be the number of successes in a binomial experiment. Then $\mu = np$.

expected number of successes

probability of success on each trial

# of trials

This formula is only true for binomial experiments

(repeated success/failure trials)

Example: Thirty percent of households in Victoria have pets. We select eight households at random.

a) Find the probability that at most two households in the sample have pets.

binomial experiment

$n = \#$ of trials $= 8$

$p = $ (a household has pets) $= 0.3$

$q = 1 - p = 0.7$

$x = \#$ of households in the sample with pets

$$P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$
$$= (8C0)(0.3)^0(0.7)^8 + (8C1)(0.3)^1(0.7)^7 + (8C2)(0.3)^2(0.7)^6$$
$$\approx 0.55$$

b) Find the probability that at least two households in the sample have pets.

$$P(x \ge 2) = 1 - P(x=0) - P(x=1)$$
$$= 1 - (8C0)(0.3)^0(0.7)^8 - (8C1)(0.3)^1(0.7)^7$$
$$\approx 0.74$$

c) Find the expected number (or mean number) of households in the sample that have pets.

$\mu = np = 2.4$ or $E(x) = 2.4$