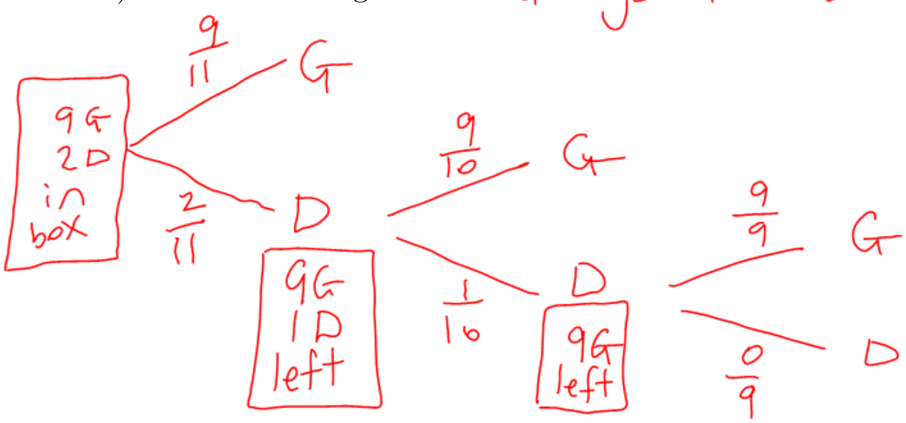
4.5 Tree Diagrams

In this section and the next one we'll explore tree diagrams, which are tools for visualizing conditional probabilities.

Example: A shipment contains nine good and two defective items. Items are selected one at a time (without replacement) until a good item is found.

a) Draw a tree diagram. Let G = good D = defective

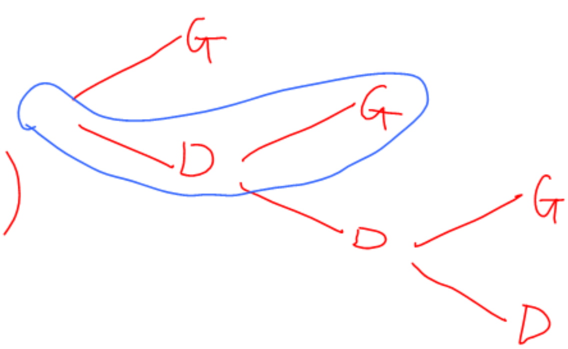


b) Find the probability that one item is selected.

$$\frac{9}{11}$$

c) Find the probability that two items are selected.

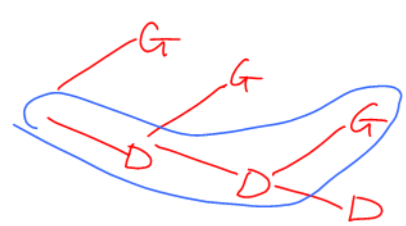
Multiply along the path.
(due to Multiplication Principle)



$$\frac{2}{11} \cdot \frac{9}{10} \approx 0.16$$

d) Find the probability that three items are selected.

Multiply along the path.



$$\frac{2}{11} \cdot \frac{1}{10} \cdot \frac{9}{9} \approx 0.02$$

Example: At a college:

$\frac{3}{5}$ of students are in Business

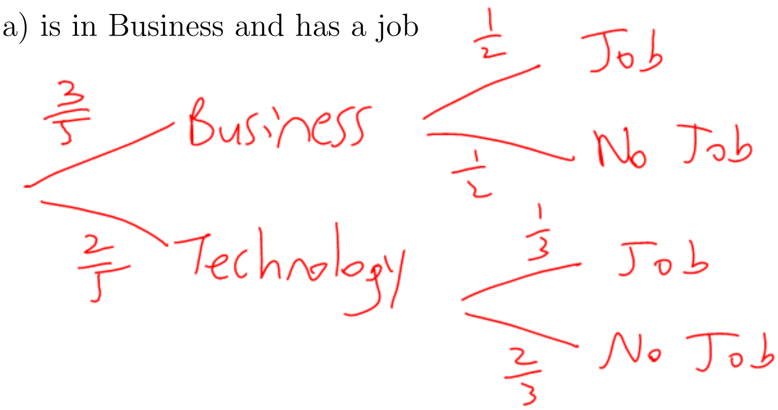
$\frac{2}{5}$ of students are in Technology

$\frac{1}{2}$ of Business students have a job

$\frac{1}{3}$ of Technology students have a job

Find the probability that a student:

a) is in Business and has a job



$$\frac{3}{5} \cdot \frac{1}{2} = 0.3$$

b) has a job

Pr(Business and Job OR Technology and Job)

$$= \frac{3}{5} \cdot \frac{1}{2} + \frac{2}{5} \cdot \frac{1}{3}$$

$$\approx 0.4333$$

AND : $\times$
OR : $+$

Multiply along each path.
Sum all paths.

c) is in Business, given that they have a job

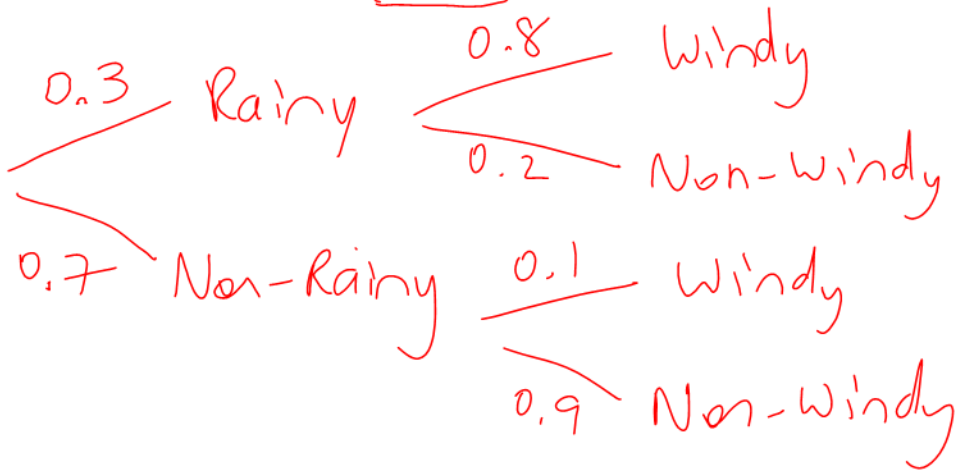
Recall $\Pr(E|F) = \dfrac{\Pr(E \cap F)}{\Pr(F)}$

$$\Pr(\text{Business} \mid \text{Job}) = \frac{\Pr(\text{Business and Job})}{\Pr(\text{Job})}$$

$$\approx \frac{0.3}{0.4333}$$

$$\approx 0.69$$

Example: 30% of days are rainy.
80% of rainy days are windy.
10% of non-rainy days are windy. ← given
Find the probability that a windy day is rainy.



$$\Pr(\text{Rainy} \mid \text{Windy})$$

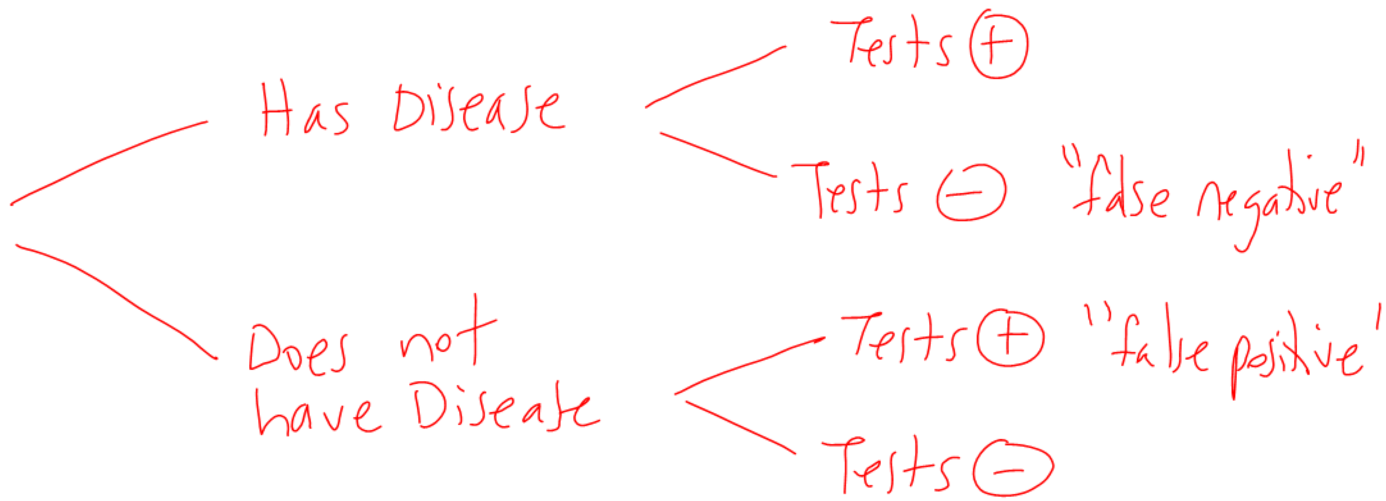
$$= \frac{\Pr(\text{Rainy} \wedge \text{Windy})}{\Pr(\text{Windy})}$$

$$= \frac{0.3(0.8)}{[0.3(0.8) + 0.7(0.1)]}$$

$$\approx 0.77$$

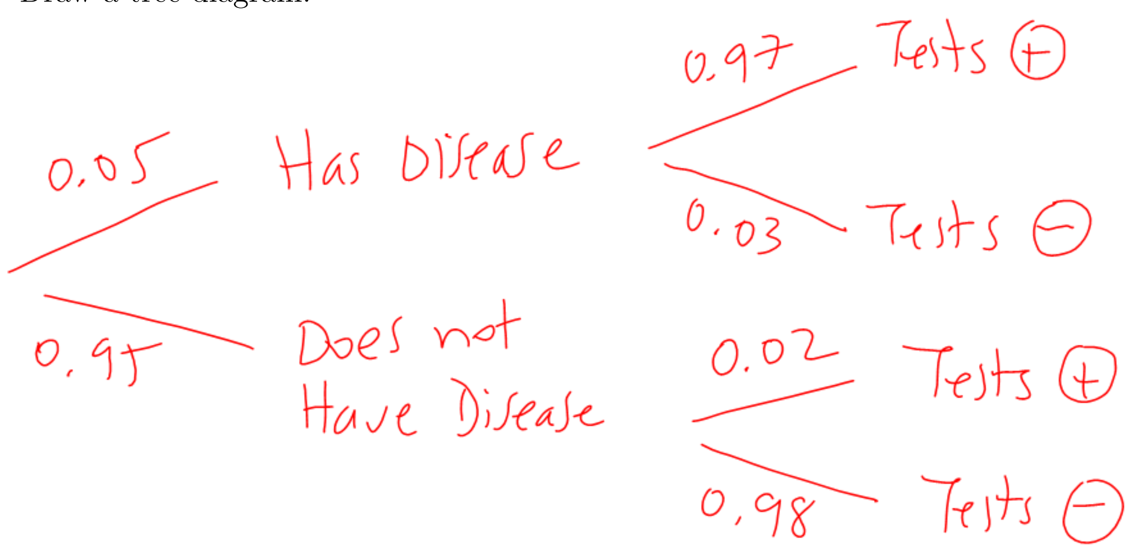
Multiply along each path.
Sum all paths.

Example: Consider the context of testing a person for a disease (for example, testing to see if they have the flu). Draw the relevant tree diagram.



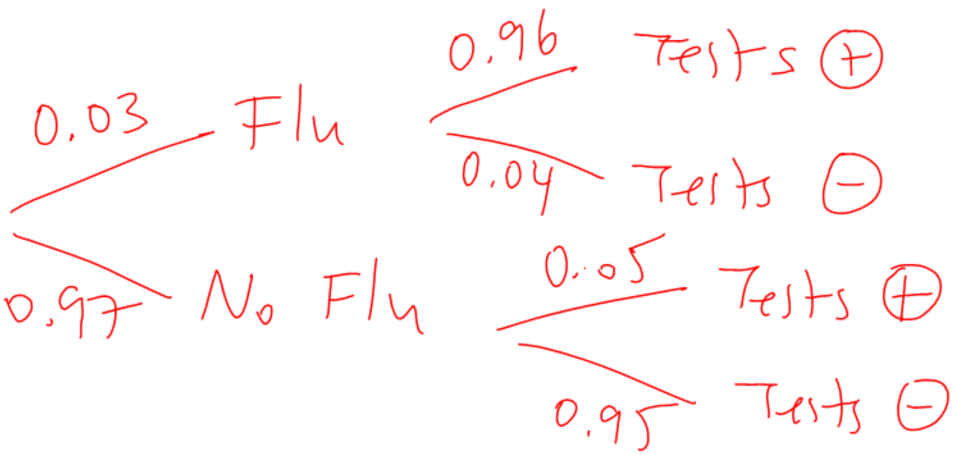
Comment: Tests can give an incorrect result due to human error, equipment error etc.

Example: 5% of patients have a certain disease.
The false-positive rate is 2%.
The false-negative rate is 3%.
Draw a tree diagram.



Example: 3% of students have the flu.
Of people with the flu, 96% test positive.
Of people not having the flu, 95% test negative.
Find the probability that a person who tests positive actually has the flu.

given



$$\Pr(\text{Flu} \mid \text{Tests } \oplus)$$

$$= \frac{\Pr(\text{Flu} \cap \text{Tests } \oplus)}{\Pr(\text{Tests } \oplus)}$$

$$= \frac{0.03(0.96)}{[0.03(0.96) + 0.97(0.05)]}$$

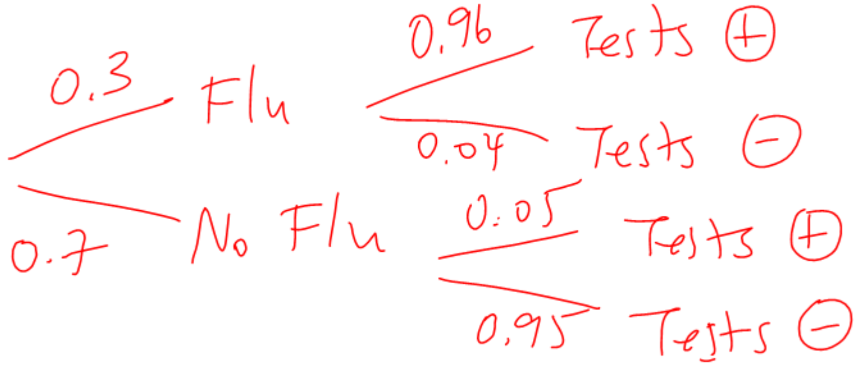
$$\simeq 0.37$$

Multiply along each path.
Sum all paths.

Comment: This probability is surprisingly low. When a disease is rare in the population, many of the positive results can be false positives.

Example: 30% of students have the flu.
Of people with the flu, 96% test positive.
Of people not having the flu, 95% test negative.
Find the probability that a person who tests positive actually has the flu.

given



$$Pr(\text{Flu} \mid \text{Tests } \oplus)$$

$$= \frac{Pr(\text{Flu} \cap \text{Tests } \oplus)}{Pr(\text{Tests } \oplus)}$$

$$= \frac{0.3(0.96)}{[0.3(0.96) + 0.7(0.05)]}$$

$$\approx 0.89$$

Multiply along each path.
Sum all paths.

Comment: We've used tree diagrams to solve the problems in this section. There is a formula called **Bayes' Theorem** that captures the same idea using algebra rather than tree diagrams.