4.3 Calculating Probabilities

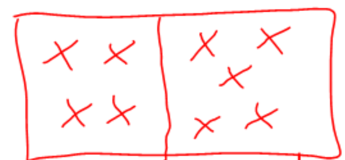
Fact: When all the outcomes of an experiment are equally likely, the probability of an event E is:

$Pr(E) = \frac{n(E)}{n(S)}$ ← # of outcomes in event E

← # of outcomes in sample space S

Example: A box contains four defective and five good items. We randomly select three items from the box. Find the probability that:

a) no defective items are selected



defective good
(9 items in total)

$n(S) =$ # of ways to select 3 of 9 items

$= C(9,3)$

$= 84$

$n(E) =$ # of ways to select 3 of 5 good items

$= C(5,3)$

$= 10$

$Pr(E) = \frac{10}{84} \simeq 0.12$

b) two defective items are selected

$n(E) = \boxed{C(4,2)}$ $\times$ $\boxed{C(5,1)}$

select 2 defective   and   select 1 good

$= 6 \times 5$

$= 30$

$Pr(E) = \frac{30}{84} \approx 0.36$

c) at least two defective items are selected

$n(E) = C(4,2) \times C(5,1) + C(4,3) \times C(5,0)$

2 def. and 1 good OR 3 def. and 0 good

$= 6 \times 5 + 4 \times 1$

$= 34$

$Pr(E) = \frac{34}{84} \approx 0.40$

Example: A fair die is rolled seven times. Find the probability of rolling exactly five 3's.

$n(S) = \boxed{6} \times \boxed{6} \times \dots \times \boxed{6}$

# of options for Roll 1

Roll 2

Roll 7

$= 6^7$

$n(E) = \boxed{C(7,5)} \times \boxed{5} \times \boxed{5}$

select 5 of 7 rolls to be a 3 (unordered)

# of options for first non-3

# of options for second non-3

$= 21 \times 5 \times 5$

$= 525$

$Pr(E) = \frac{525}{6^7} \approx 0.002$

Example: A five-card poker hand is dealt. Find the probability of getting two pairs. For example, the hand could be 88QQK or 33772.

$$n(S) = \boxed{C(52,5)}$$

Select 5 of 52 cards (unordered)

$$= 2,598,960$$

$$n(E) = \boxed{C(13,2)} \times \boxed{C(4,2)} \times \boxed{C(4,2)} \times \boxed{44}$$

Select 2 of 13 denominations for pairs

Select suits for 1^{st} pair

Select suits for 2^{nd} pair

Fifth card must be a different denomination $52-4-4=44$

$$= 78 \times 6 \times 6 \times 44$$

$$= 123,552$$

$$Pr(E) = \frac{123,552}{2,598,960} \approx 0.05$$

Recall the Complement Rule:
$Pr(E) = 1 - Pr(E\,')$

Comment: This is true because $Pr(E) + Pr(E\,') = 1$.
The Complement Rule can also be rephrased as: $Pr(E\,') = 1 - Pr(E)$

Example: In a class of 40 students, six have the flu. Five students are randomly selected from the class. Find the probability that at least one of them has the flu.

E : at least one has the flu
E' : none have the flu

6	34
Flu	No Flu

$n(S) = C(40,5)$
$= 658{,}008$

$n(E') = C(34,5)$
$= 278{,}256$

$Pr(E') = \dfrac{278256}{658008} \approx 0.42$

$Pr(E) = 1 - Pr(E')$
≈ 0.58

Note: $Pr(E)$ is hard to calculate directly.

Example: A group of 20 people are randomly selected. Find the probability that at least two of them are born on the same day of the year. We'll assume 365 days in a year (meaning ignore February 29).

E: at least two are born on same day

E': all born on different days

$$n(S) = \underset{\substack{\text{\# of options}\\ \text{for Person 1's}\\ \text{birthday}}}{\boxed{365}} \times \underset{\text{Person 2}}{\boxed{365}} \times \cdots \times \underset{\text{Person 20}}{\boxed{365}}$$

$$= 365^{20}$$

$$n(E') = 365 \times 364 \times \cdots$$
$$= P(365, 20)$$

$$Pr(E') = \frac{P(365, 20)}{365^{20}} \approx 0.59$$

$$Pr(E) = 1 - Pr(E')$$
$$\approx 0.41$$

Note: $Pr(E)$ is hard to calculate directly.