4.2 Basic Probability Concepts

Definition: The **probability** of an event is a measure of how likely the event is.

Notation: We often write probabilities as decimals. $Pr(E) = 0.35$ means that the probability of event E is 35%.

Fact: Any probability is always between 0 and 1 inclusive.
$Pr(F) = 0$ means the probability of event F is 0%, in other words event F cannot happen.
$Pr(G) = 1$ means the probability of event G is 100%, in other words event G is guaranteed to happen.

Fact: For any experiment, if we sum the probabilities of all the outcomes we get 1.

Definition: A **probability distribution** is a table that lists the different outcomes of an experiment and their probabilities.

Example: We toss a fair coin and record heads or tails. Let's write down the probability distribution.

Outcome	Probability
H	0.5
T	0.5

Example: Some students are polled on their program. Use the following information to find the probability distribution.

Program	Number of Students
Business	13
Technology	18
Nursing	9

Total = 40

Outcome	Probability
Business	$\frac{13}{40} = 0.325$
Technology	$\frac{18}{40} = 0.45$
Nursing	$\frac{9}{40} = 0.225$

Fact: The probability of an event is the sum of the probabilities of the relevant outcomes.

Example: An unfair four-sided die has the following probability distribution:

Roll	Probability
1	0.1
2	0.35
3	0.3
4	0.25

a) Find the probability that a roll is less than 3.

Pr(roll is 1) + Pr(roll is 2)

$= 0.1 + 0.35$

$= 0.45$

b) Find the probability that a roll is odd.

Pr(roll is 1) + Pr(roll is 3)

$= 0.1 + 0.3$

$= 0.4$

c) Find the probability that a roll is $\underbrace{\text{less than 2 and even}}_{\text{impossible}}$.

Pr(roll is less than 2 and even) = 0

Example: An experiment has possible outcomes A, B and C. We are given $Pr(A) = 0.4$ and we are told that outcome B is three times as likely as outcome C. Find the probability distribution.

Let $Pr(C) = x$

$Pr(B) = 3x$

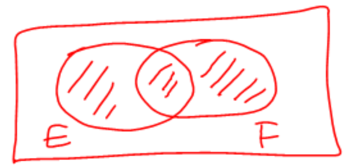
Outcome	Probability
A	0.4
B	$3x$
C	x

$$0.4 + 3x + x = 1$$

$$4x = 0.6$$

$$x = 0.15$$

Outcome	Probability
A	0.4
B	0.45
C	0.15



$E \cup F$

Fact: Inclusion-Exclusion Principle
$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$

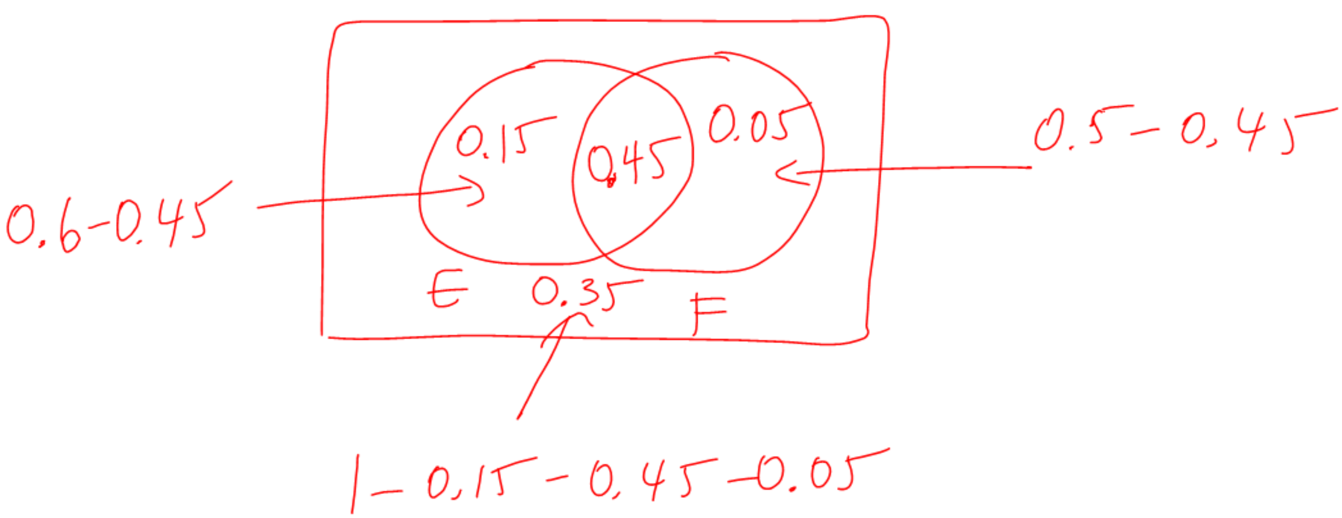
Comment: Compare this with Section 3.2:
$n(E \cup F) = n(E) + n(F) - n(E \cap F)$

Example: Given $Pr(E) = 0.6, Pr(F) = 0.5$ and $Pr(E \cup F) = 0.65$.

a) Find $Pr(E \cap F)$

$$Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F)$$
$$0.65 = 0.6 + 0.5 - Pr(E \cap F)$$
$$-0.45 = -Pr(E \cap F)$$
$$0.45 = Pr(E \cap F)$$

b) Draw a Venn diagram



c) Find $Pr(E\,' \cap F)$

(not E) and F

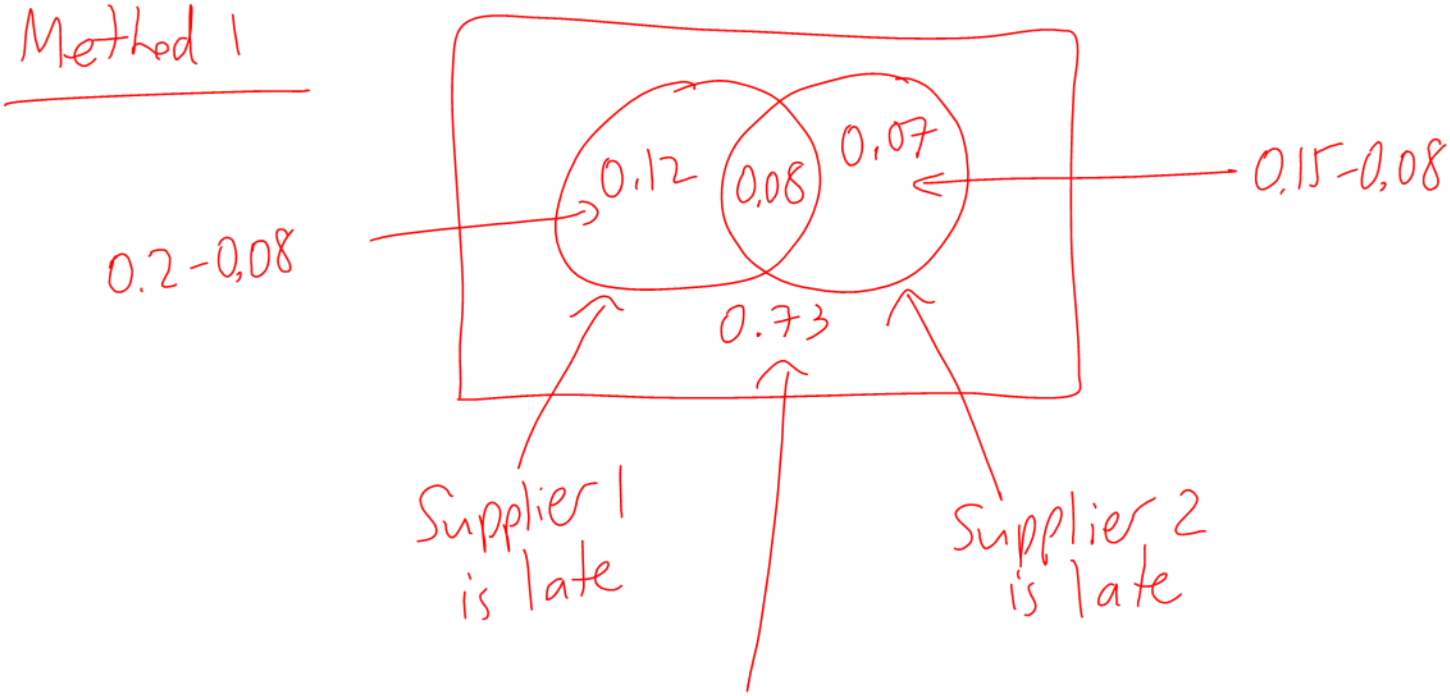
$= 0.05$

Fact: Complement Rule
$Pr(E) = 1 - Pr(E\,')$

Comment: This is true because $Pr(E) + Pr(E\,') = 1$.
The Complement Rule can also be rephrased as: $Pr(E\,') = 1 - Pr(E)$

Example: A company has two suppliers. The probability that Supplier 1 is late is 20%. The probability that Supplier 2 is late is 15%. The probability that both suppliers are late is 8%. Find the probability that neither supplier is late.

Method 1



$1 - 0{,}12 - 0{,}08 - 0{,}07$

Pr(neither supplier is late) $= 0{,}73$

Example Continued...

Method 2

Pr(Supplier 1 or Supplier 2 is late)

= Pr(Supplier 1 is late) + Pr(Supplier 2 is late) − Pr(both are late)

= 0.2 + 0.15 − 0.08

= 0.27

Pr(neither is late)

= 1 − Pr(Supplier 1 or Supplier 2 is late)

= 1 − 0.27

= 0.73