4.1 Sample Space and Events

In this chapter we'll consider whether specific events are likely or unlikely.

Definition: The **sample space** is the set of possible outcomes of an experiment. An **event** is a subset of the sample space.

Example: Suppose we roll a (six-sided) die. Let's write out the sample space and some possible events.

experiment

Sample space $S = \{1, 2, 3, 4, 5, 6\}$

Some possible events:

E: roll an even number $E = \{2, 4, 6\}$

F: roll is less than 3 $F = \{1, 2\}$

Example: We toss a coin three times and record heads or tails.

a) Write out the sample space.

$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$

b) Let E: at most one tail. Write out E.

$E = \{HHH, HHT, HTH, THH\}$

Example: A small team has four employees named Al, Bo, Cindy and Dan. We want to select two of them for a project.

a) Write out the sample space.

$S = \{ \{Al, Bo\}, \{Al, Cindy\}, \{Al, Dan\}, \{Bo, Cindy\}, \{Bo, Dan\}, \{Cindy, Dan\} \}$

b) Let E: Al is not chosen. Write out E.

$E = \{ \{Bo, Cindy\}, \{Bo, Dan\}, \{Cindy, Dan\} \}$

Example: Suppose an experiment has sample space $S = \{1, 2\}$. Write out all the possible events.

Write out all possible subsets of S.

$\{1, 2\}$

$\{1\}$

$\{2\}$

ϕ

Example: We roll a red die and a blue die.

a) How many outcomes are there in the sample space?

6 × 6 = 36

# of options for red die

# of options for blue die

To visualize the 36 outcomes:

Red \ Blue	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

b) Let E: the rolls sum to six. Write out E.

$E = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

red blue

red blue

Example: We roll a red die and a blue die.
Let F: the roll on the red die is 4.
Let G: the roll on the red die is 3 or 4, and the roll on the blue die is at least 5.

a) Write out F.

$F = \{(4,1), (4,2), \ldots, (4,6)\}$

red blue red blue

b) Write out G.

$G = \{(3,5), (3,6), (4,5), (4,6)\}$

red blue red blue

c) Find $F \cup G$. Recall $F \cup G$: F or G or both

$F \cup G = \{(3,5), (3,6), (4,1), (4,2), \ldots, (4,6)\}$

d) Find $F' \cap G$. ← means (not F) and G

$F' \cap G = \{(3,5), (3,6)\}$

Definition: Two events E and F are **mutually exclusive** if $E \cap F = \varnothing$.

Comment: Here are two ways of rephrasing "E and F are mutually exclusive":
"E and F have no outcomes in common"
"E and F can't happen at the same time"

Example: We flip a coin three times. Are the following events mutually exclusive?

a) E: No heads appear.
F: No tails appear.

$E = \{TTT\}$

$F = \{HHH\}$

$E \cap F = \phi$

Yes, E and F are mutually exclusive.

b) E: At most one tail appears.
F: No tails appear.

$E = \{HHH, THH, HTH, HHT\}$

$F = \{HHH\}$

$E \cap F \neq \phi$

No, E and F are not mutually exclusive.