

3.4 The Multiplication Principle

In this section and the next two sections we'll look at how many ways there are to perform a certain activity. This will prepare us for the concept of probability in Chapter 4.

Fact: The Multiplication Principle

When performing a sequence of tasks, the number of possibilities is multiplied.

Example: There are six different routes from City A to City B, and two different routes from City B to City C. How many possible routes are there from City A to City C?



$$\boxed{6} \times \boxed{2} = 12$$

of routes from A to B # of routes from B to C

Example: How many possible outcomes are there if we toss a coin four times?

e.g. $\begin{array}{cccc} H & H & H & H \\ T & H & H & T \\ T & T & T & H \end{array}$

$$\boxed{2} \times \boxed{2} \times \boxed{2} \times \boxed{2} = 16$$

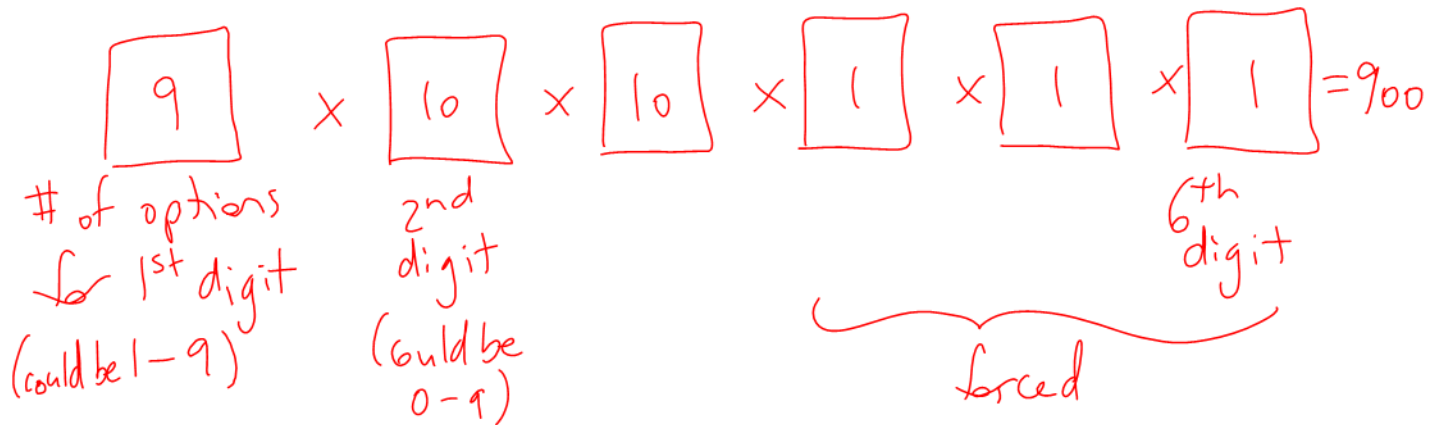
of options for 1st toss (H or T) 2nd toss 3rd toss 4th toss

3.4 The Multiplication Principle

Example: How many six-digit palindromes are there?

e.g. 784487
222222

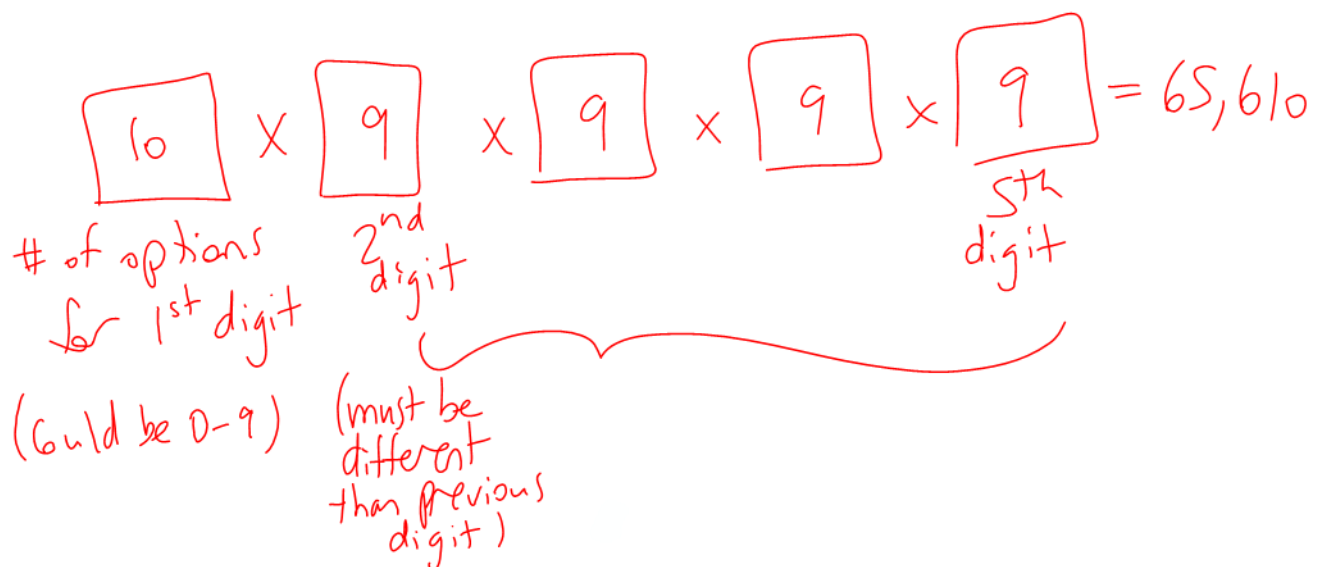
Caution: 019910 is not allowed
because this is written 19910



Example: Count the number of five-digit passcodes using digits 0 to 9 if adjacent digits can't be the same.

e.g. 13234
98989
01010

Caution: 13324 is not allowed



3.4 The Multiplication Principle

Example: How many ten-digit phone numbers have area codes 250, 778 or 236 such that the last seven digits aren't 000-0000?

$$\# \text{ of area codes} = 3$$

$$\# \text{ of seven-digit phone numbers} = 10 \times 10 \times \dots \times 10 \\ = 10^7$$

$$\# \text{ of seven-digit phone numbers other than } 000-0000 \\ = 10^7 - 1$$

$$\begin{aligned} \text{Total } \# &= \boxed{3} \times \boxed{10^7 - 1} \\ &= 3(10^7 - 1) \\ &= 29,999,997 \end{aligned}$$

Example: How many ways are there to arrange seven books on a shelf from left to right?

$$\begin{array}{ccccccc} \boxed{7} & \times & \boxed{6} & \times & \boxed{5} & \times & \boxed{4} & \times & \boxed{3} & \times & \boxed{2} & \times & \boxed{1} \\ \# \text{ of options} & & \text{2nd} & & & & & & & & \text{7th} & & \\ \text{for 1st book} & & \text{book} & & & & & & & & \text{book} & & \end{array}$$

$$= 5,040$$

3.4 The Multiplication Principle

Example: Ten people interview for a job.

a) How many ways are there to choose the best and second-best candidate?

$$\boxed{10} \times \boxed{9} = 90$$

of ways to choose best 2nd best

b) How many ways are there to rank all ten people from 1st to 10th?

$$\boxed{10} \times \boxed{9} \times \boxed{8} \times \dots \times \boxed{1} = 3,628,800$$

of ways to choose 1st 2nd 3rd 10th

Example: How many three-letter “words” (including nonsense words) can be formed from A, B, C if:

a) repetition is allowed

e.g. AAA
CBB

$$\boxed{3} \times \boxed{3} \times \boxed{3} = 27$$

of options for 1st letter 2nd letter 3rd letter

b) repetition is not allowed

$$\boxed{3} \times \boxed{2} \times \boxed{1} = 6$$

of options for 1st letter 2nd letter 3rd letter

ABC	BAC	CAB
ACB	BCA	CBA