3.2 The Inclusion-Exclusion Principle

Notation: $n(S)$ means the number of elements in set S.

Example: Let $A = \{a, b, c\}$ and $B = \varnothing$. Find $n(A)$ and $n(B)$.

$n(A) = 3$

$n(B) = 0$

Fact: The Inclusion-Exclusion Principle.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Example: Confirm the Inclusion-Exclusion Principle for $A = \{w, x, y\}$ and $B = \{x, y, z\}$.

$A \cup B = \{w, x, y, z\}$

$A \cap B = \{x, y\}$

$n(A \cup B) = 4$

$n(A) + n(B) - n(A \cap B)$

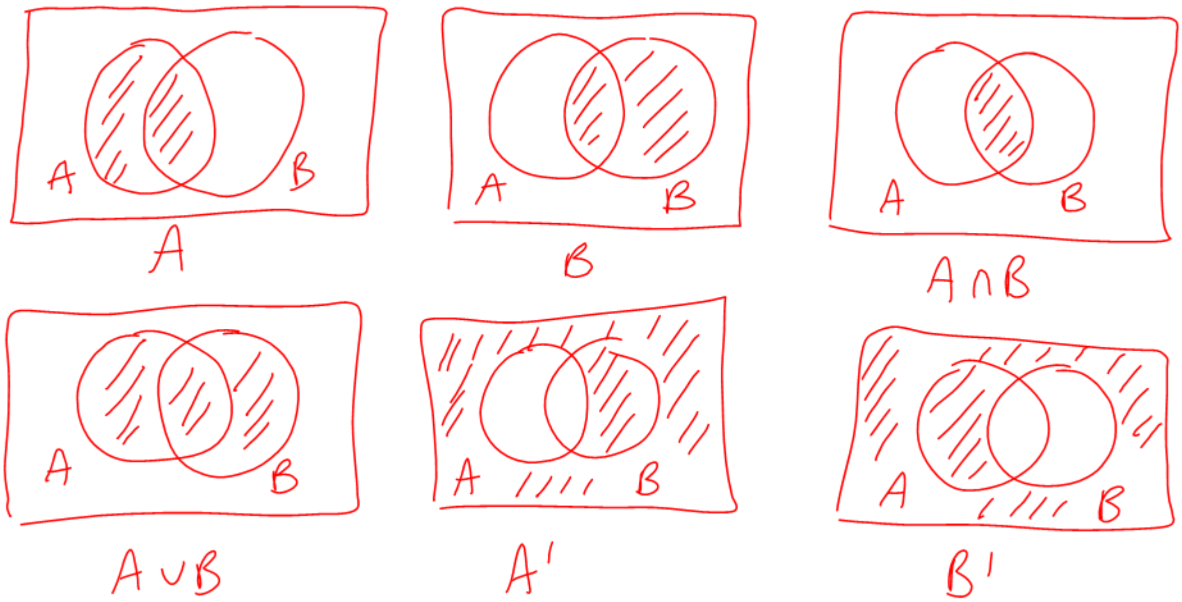
$= 3 + 3 - 2$

$= 4$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ✓

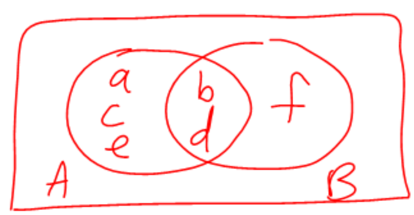
Definition: A **Venn diagram** is a way to visualize different sets.

Example: Let's draw Venn diagrams for A, B, $A \cap B$, $A \cup B$, A' and B'.

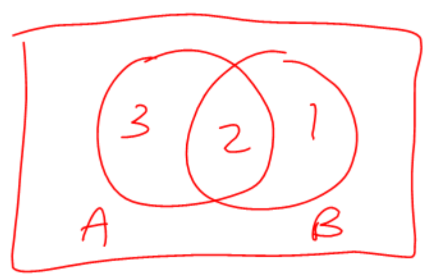


Example: Let $A = \{a, b, c, d, e\}$ and $B = \{b, d, f\}$. Draw a Venn diagram for A and B showing the elements. Draw another Venn diagram for A and B showing the number of elements. Then confirm the Inclusion-Exclusion Principle for A and B.

Elements:



Number of elements:



$n(A \cup B) = 6$

$n(A) + n(B) - n(A \cap B) = 5 + 3 - 2 = 6$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ ✓

Example: A company has 300 employees: 275 are full-time and 230 are permanent, while 285 are full-time or permanent. How many are full-time and permanent?

Let F = full-time employees
P = permanent "

$$n(F \cup P) = n(F) + n(P) - n(F \cap P)$$

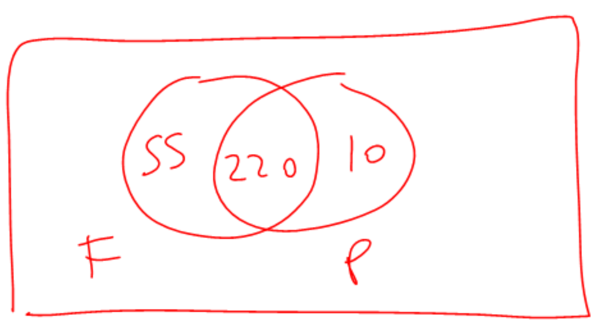
$$285 = 275 + 230 - n(F \cap P)$$

$$285 - 275 - 230 = -n(F \cap P)$$

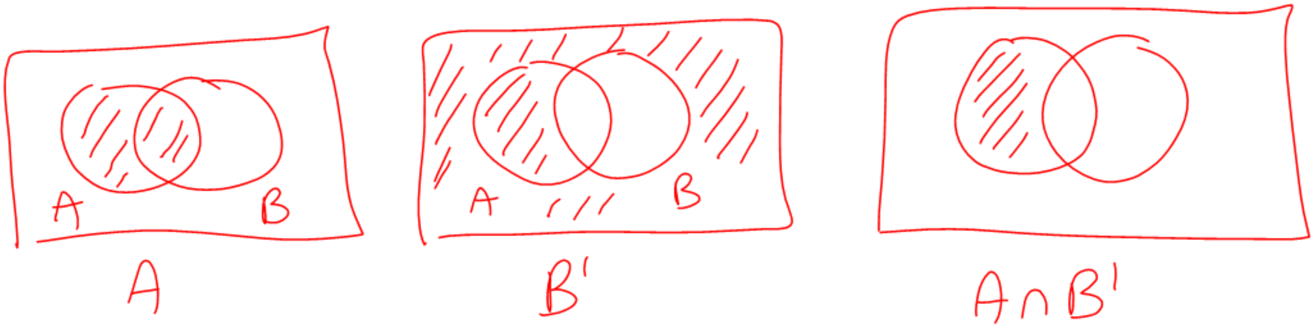
$$-220 = -n(F \cap P)$$

$$220 = n(F \cap P)$$

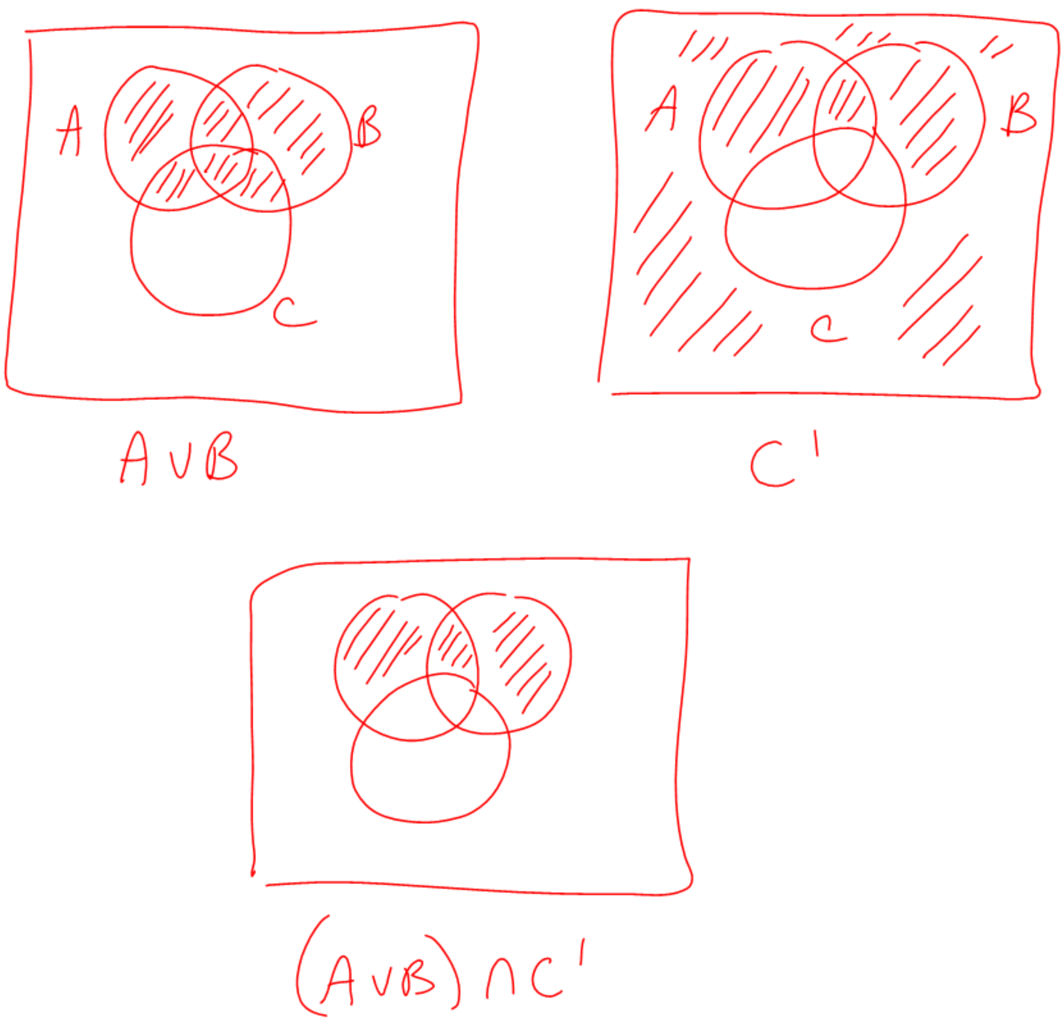
Optional: Venn diagram



Example: Draw a Venn diagram for $A \cap B'$.



Example: Draw a Venn diagram for $(A \cup B) \cap C\,'$.



Fact: De Morgan's Laws.

Caution: Operations Reverse

For any sets S and T:
$(S \cup T)' = S\,' \cap T\,'$
$(S \cap T)' = S\,' \cup T\,'$

Example: Use Venn diagrams to confirm that $(S \cap T)' = S\,' \cup T\,'$.

Example: Simplify $(A \cup B')'$.

$= A' \cap B''$
$= A' \cap B$

Comment: The last example shows that there can be multiple ways to describe a given set.