1.3 Intersection of Two Lines

Example: Find the intersection of the lines below. In other words, solve the system of equations.

$$\begin{aligned} 3x + 2y &= 2 \\ 9x - 2y &= 18 \end{aligned}$$

$2y = 2 - 3x$

$y = 1 - \frac{3}{2}x$

$-2y = 18 - 9x$

$y = -9 + \frac{9}{2}x$

$y = y$

$1 - \frac{3}{2}x = -9 + \frac{9}{2}x$

Multiply by 2: $2 - 3x = -18 + 9x$

$-12x = -20$

$x = \frac{-20}{-12} = \frac{5}{3}$

$x = \frac{5}{3} \to$ either equation

$x = \frac{5}{3} \to \quad y = 1 - \frac{3}{2}x$

$y = 1 - \frac{3}{2}\left(\frac{5}{3}\right)$

$y = \frac{2}{2} - \frac{5}{2}$

$y = -\frac{3}{2}$

$\boxed{(x,y) = \left(\frac{5}{3}, -\frac{3}{2}\right)}$

We'll stick to this method in class.
Feel free to use other methods (like substitution) if you prefer.

Example: Find the intersection of the lines below. In other words, solve the system of equations.

$$\begin{aligned} x &= -1 \\ y &= -3x+2 \end{aligned}$$

$\leftarrow$ no y in this equation

$x = -1 \rightarrow y = -3x + 2$

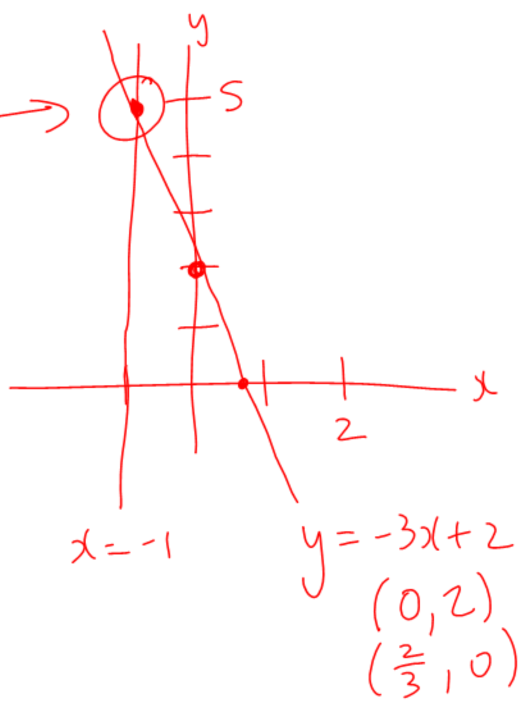
$y = 3 + 2$

$y = 5$

$$\boxed{(x,y) = (-1,5)}$$

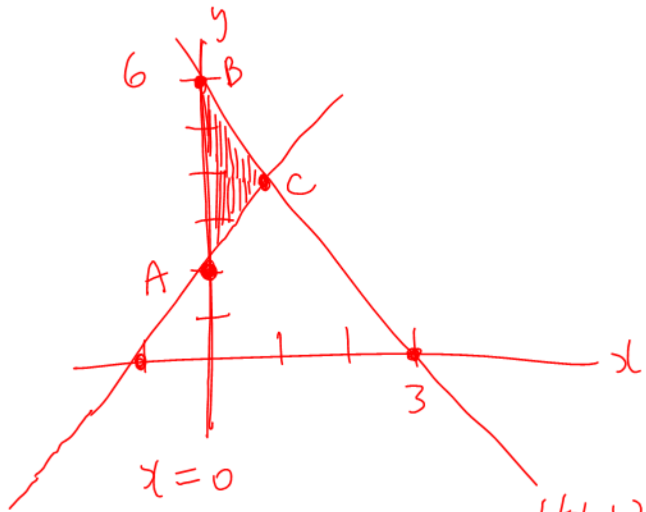
Aside:

$(x,y) = (-1,5) \rightarrow$



Example: Graph the lines $x = 0$, $-2x + y = 2$ and $4x + 2y = 12$. Find the vertices of the triangle formed by the three lines.

corners



$4x + 2y = 12$
$(3, 0)$
$(0, 6)$

$-2x + y = 2$
$(0, 2)$
$(-1, 0)$

$$A: \begin{cases} x = 0 \\ -2x + y = 2 \end{cases}$$

$x = 0 \rightarrow -2x + y = 2$

$y = 2$

$A = (0, 2)$

$$B: \begin{cases} x = 0 \\ 4x + 2y = 12 \end{cases}$$

$x = 0 \rightarrow 4x + 2y = 12$

$2y = 12$

$y = 6$

$B = (0, 6)$

$\longrightarrow$

Example Continued...

$$C: \begin{cases} 4x + 2y = 12 \\ -2x + y = 2 \end{cases}$$

$$2y = 12 - 4x$$
$$y = 6 - 2x$$

$$y = 2 + 2x$$

$$y = y$$
$$6 - 2x = 2 + 2x$$
$$4 = 4x$$
$$1 = x$$

$x = 1 \rightarrow$ either equation

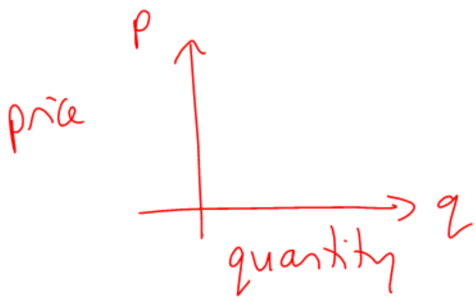
$x = 1 \rightarrow$ $y = 2 + 2x$

$$y = 4$$

$$C = (1, 4)$$

Let's look at the concept of supply and demand in business. We will graph the price of an item (written p) as a function of the quantity (written q).

Example: Draw the quantity axis and the price axis.

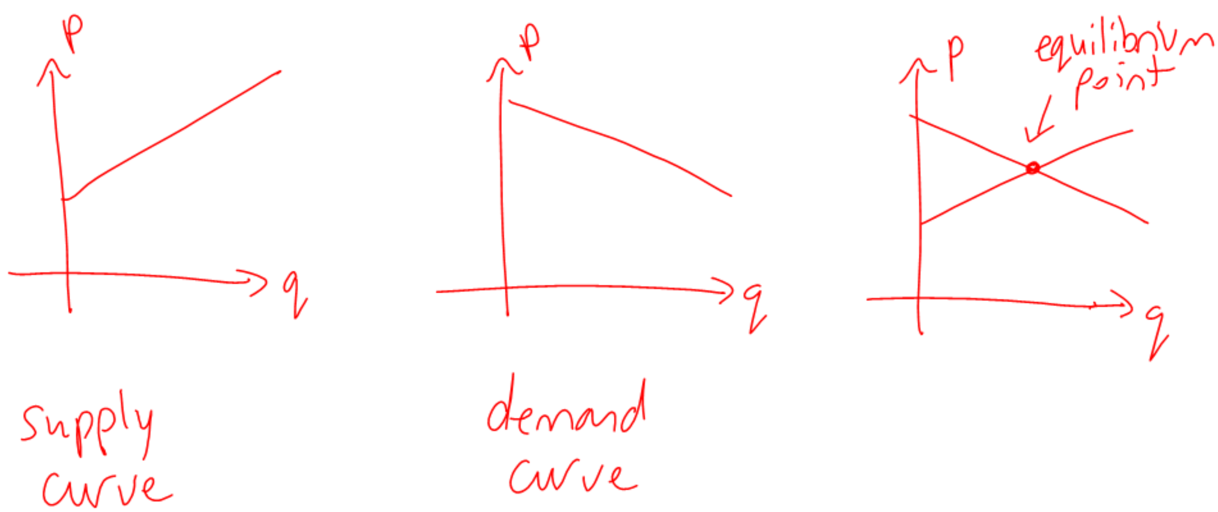


Definition: The supply curve is a line. It shows the relationship between the price of an item and the quantity that producers are willing to make.

Definition: The demand curve is a line. It shows the relationship between the price of an item and the quantity that consumers are willing to purchase.

Definition: The equilibrium point is the point where the supply curve and the demand curve intersect.

Example: Draw a typical supply curve, a typical demand curve, and the equilibrium point.



Example: We are given a supply curve and a demand curve. The price p is in dollars and the quantity q is in thousands of units. Find the equilibrium quantity and price.

$$\begin{aligned} p &= 0.005q + 2.5 \\ p &= -0.002q + 6.7 \end{aligned}$$

$$p = p$$

$$0.005q + 2.5 = -0.002q + 6.7$$

$$0.007q = 4.2$$

$$q = 600$$

$q = 600 \rightarrow$ either equation

$q = 600 \rightarrow p = 0.005q + 2.5$

$p = 5.5$

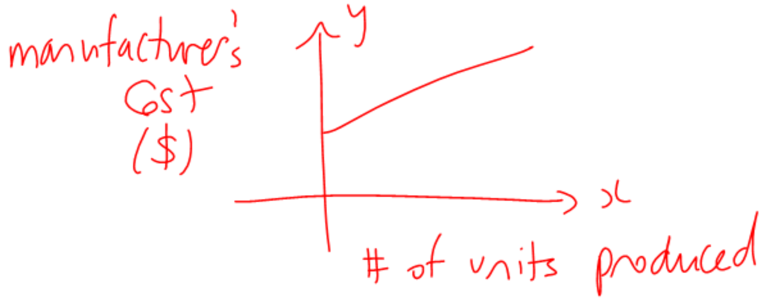
Equilibrium quantity: 600,000 units

" price: $5.50

(Supply equals demand)

Fact: A cost line has the form $y = mx + b$, where y is the cost in dollars and x is the number of units produced. The number m is called the **marginal cost** or the **cost per unit**, and the number b is called the **fixed cost**.

Example: Graph a typical cost line.



Example: Manufacturer A can produce a product for \$300 plus \$10 per unit. Manufacturer B can produce a product for \$200 plus \$12 per unit. How many units result in the costs being equal? What is the cost for this number of units?

Cost line: $y = mx + b$ ← fixed cost

marginal cost

Manufacturer A: $y = 10x + 300$

" B: $y = 12x + 200$

Find the intersection of the cost lines.

$$y = y$$
$$10x + 300 = 12x + 200$$
$$100 = 2x$$
$$50 = x$$

→

Example Continued...

$x = 50 \rightarrow$ either equation

$x = 50 \rightarrow \quad y = 10x + 300$

$y = 800$

When they produce 50 units, both manufacturers have a cost of $800.

Aside: