1.2 Linear Inequalities

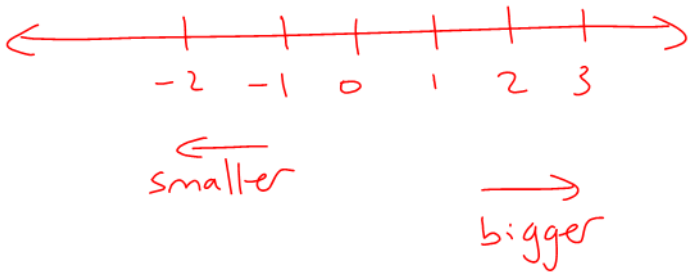
Example: Let's write down some inequalities:

$3 \geq 2$ $\qquad$ $2 \leq 3$

$3 \geq 3$ $\qquad$ $3 \leq 3$

$-1 \geq -2$ $\qquad$ $-2 \leq -1$

Example: Let's draw the number line to help visualize these inequalities:



Fact: We can add any number to both sides of an inequality.
We can subtract any number from both sides of an inequality.

Fact: We can multiply both sides of an inequality by any nonzero number, but **multiplying by a negative number reverses the inequality**.

$-0.25 \geq -0.5$

Multiply by -4: $1 \leq 2$

Fact: We can divide both sides of an inequality by any nonzero number, but **dividing by a negative number reverses the inequality**.

$-6 \leq -3$

Divide by -3: $2 \geq 1$

Example: Solve $6 - 2x \geq 8$.

Subtract 6: $-2x \geq 2$

Divide by -2: $x \leq -1$ ← Inequality reverses

Definition: The **standard form** for an inequality is: $y \leq mx + b$ or $y \geq mx + b$.

Example: a) Put $8x - 4y \geq 12$ in standard form.

$-4y \geq -8x + 12$

Divide by -4: $\frac{-4y}{-4} \leq \frac{-8x}{-4} + \frac{12}{-4}$

$y \leq 2x - 3$

b) Does $(0, 0)$ satisfy $y \leq 2x - 3$?

Sub $x=0, y=0$: $0 \leq 0 - 3$?

$0 \leq -3$?

No

c) Does $(2, -1)$ satisfy $y \leq 2x - 3$?

Sub $x=2, y=-1$: $-1 \leq 2(2) - 3$?

$-1 \leq 1$?

YES

Example: Put $0.3x - 0.4y \leq 2$ in standard form.

Multiply by 10: $3x - 4y \leq 20$

$-4y \leq -3x + 20$

Divide by -4: $\frac{-4y}{-4} \geqslant \frac{-3x}{-4} + \frac{20}{-4}$

$y \geqslant \frac{3x}{4} - 5$

Example: Put $\frac{2}{7}x - 3y \leq \frac{4}{7}$ in standard form.

Multiply by 7: $2x - 21y \leq 4$

$-21y \leq -2x + 4$

Divide by -21: $\frac{-21y}{-21} \geqslant \frac{-2x}{-21} + \frac{4}{-21}$

$y \geqslant \frac{2x}{21} - \frac{4}{21}$

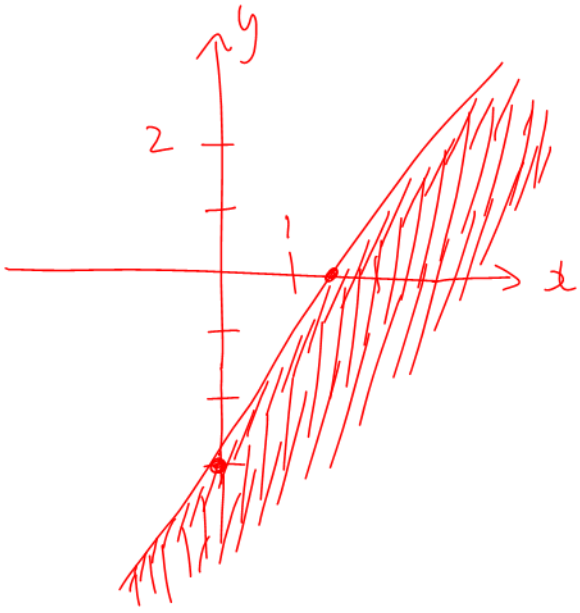
Fact: To graph an inequality, we first graph the associated line. All the points that satisfy the inequality will be on one side of the line. To figure out which side, we test any point that is not on the line.

Example: Graph $8x - 4y \geq 12$.

associated line : $8x - 4y = 12$

Sub $x = 0$: $-4y = 12$, $y = -3$, $(0, -3)$

Sub $y = 0$: $8x = 12$, $x = \frac{12}{8} = \frac{3}{2}$, $(\frac{3}{2}, 0)$



The shaded points satisfy the inequality.

Test any point that is not on the line, say $(0,0)$.

Sub $x = 0$, $y = 0$ $\rightarrow$ $8x - 4y \geq 12$

$0 \geq 12$?

No

Example: Graph $-3x + y \geq 0$.

$-3x+y=0$

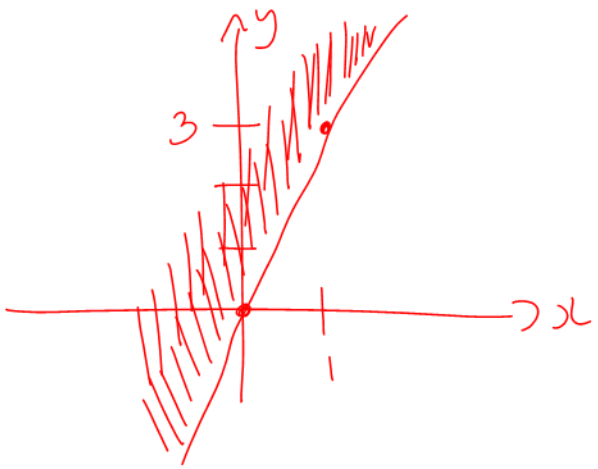
$x=0$: $y=0$ $(0,0)$

$y=0$: $x=0$ $(0,0)$

Need another point. Sub any nonzero x-value.

$x=1$: $-3+y=0$

$y=3$ $(1,3)$



Test any point that is not on the line, say $(1,0)$.

Sub $x=1$, $y=0$ $\rightarrow$ $-3x+y \geq 0$

$-3 \geq 0$?

No

Example: Graph the feasible set for the system of inequalities:

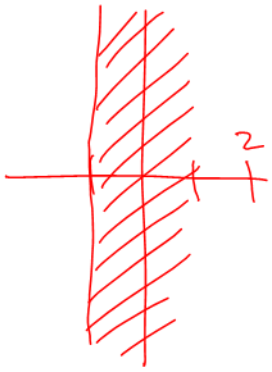
$$\begin{aligned} x+1 &\geq 0 \\ x+y &\leq 10 \\ -x+0.5y &\geq 0.5 \end{aligned}$$

$x+1 \geq 0$

$x+1 = 0$

$x = -1$

vertical line



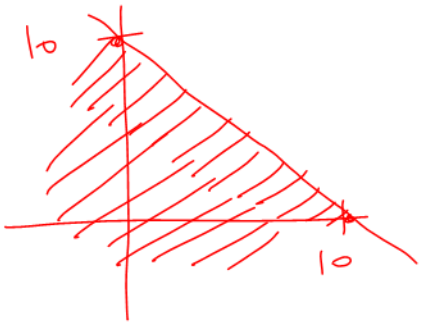
Test (0,0): YES

$x+y \leq 10$

$x+y = 10$

(0, 10)

(10, 0)



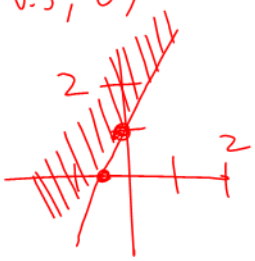
Test (0,0): YES

$-x+0.5y \geq 0.5$

$-x+0.5y = 0.5$

(0, 1)

(-0.5, 0)

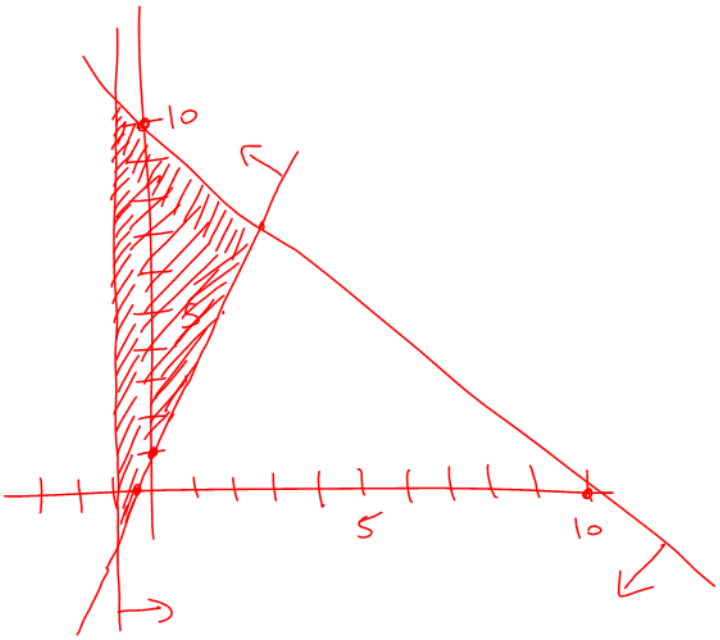


Test (0,0): No

Continued →

Example Continued...

All together:



The shaded points satisfy all the inequalities.

Example: Graph the feasible set for the system of inequalities:

$$\begin{aligned} x + y &\leq 6 \\ -2x + y &\geq 0 \\ y &\geq 2 \\ x &\geq 0 \end{aligned}$$

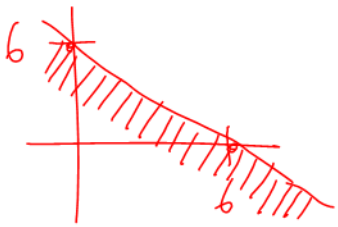
$x+y \leq 6$

$x+y=6$

$(0,6)$

$(6,0)$

Test $(0,0)$: YES



$-2x+y \geq 0$

$-2x+y=0$

$(0,0)$

Need another point.

$x=1 \rightarrow -2x+y=0$

$-2+y=0$

$y=2$

$(1,2)$

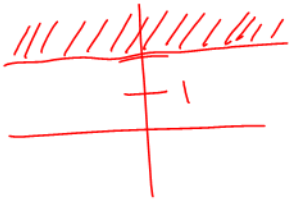
Test $(1,0)$: No

$y \geq 2$

$y=2$

horizontal line

Test $(0,0)$: No



$x \geq 0$

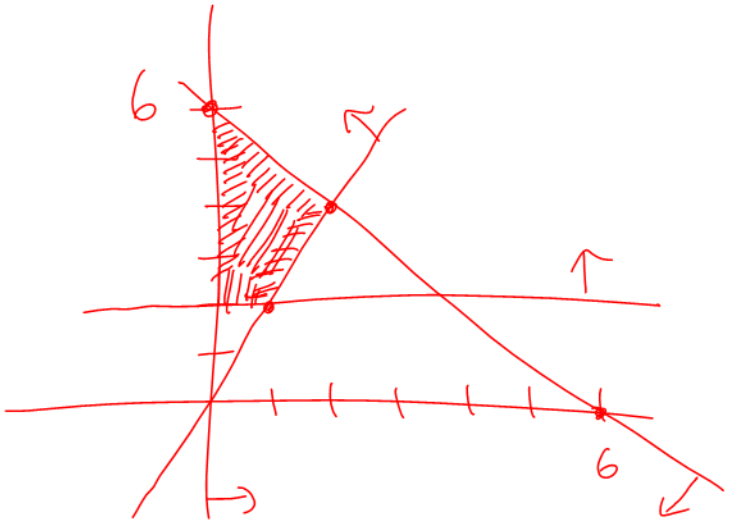
$x=0$

vertical line

Test $(1,0)$: YES

Example Continued...

All together:



Note:
Answers at the back of the Suggested HW pdf have incorrect shading. Solutions on the website are correct.