2.1 and 2.2 Linear Programming

Suppose a manufacturing company has a fixed amount of raw materials and wants to maximize their profit. In this chapter we'll use our understanding of equations and inequalities to maximize or minimize a quantity with some restrictions. This process is called **linear programming**.

Fact: The Fundamental Theorem
The maximum (or minimum) value of the objective function occurs at one of the vertices of the feasible set.

whatever we are maximizing or minimizing

Corners

Comment: Each linear programming problem will be divided into eight short steps. We'll revisit the fact above during the seventh step.

Example: Each day a company has 60kg of wood and 100kg of metal available. A chair uses 2kg of wood, 4kg of metal and yields a profit of \$14. A table uses 3kg of wood, 4kg of metal and yields a profit of \$20. How many chairs and tables maximize the daily profit?

1) Variables

Let x = # of chairs produced each day
y = " tables "

2) Chart

	Chair (x)	Table (y)	Available
Wood (kg)	2	3	60
Metal (kg)	4	4	100
Profit (\$)	14	20	//////////

3) Inequalities

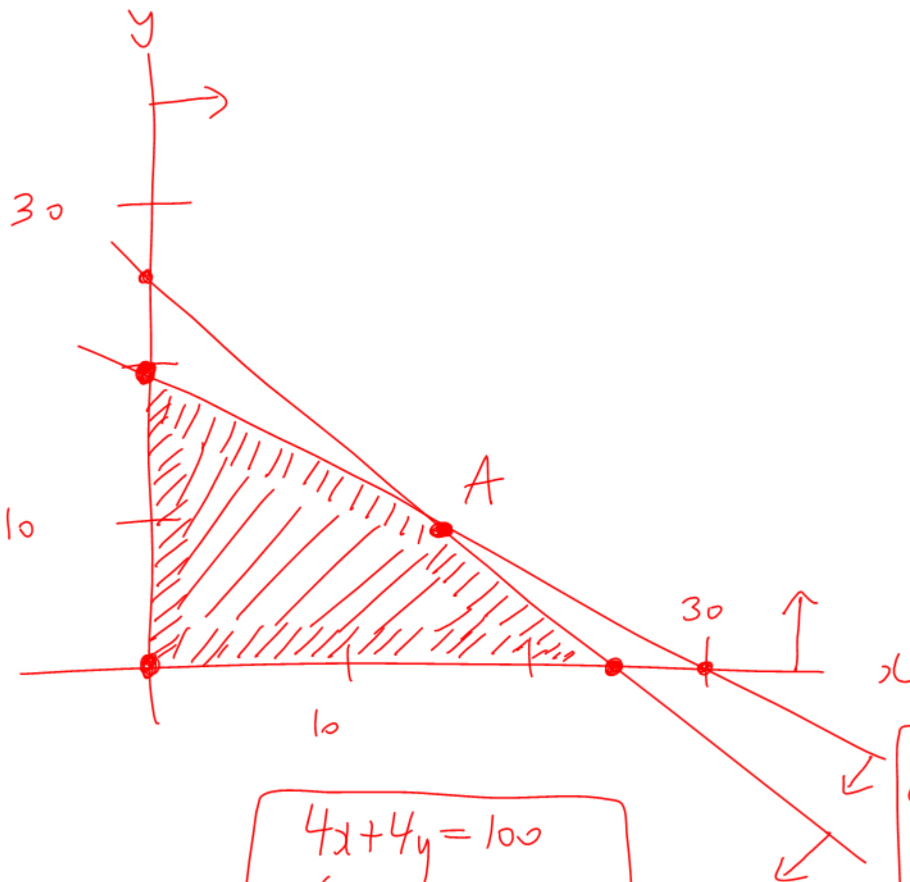
Wood (kg): $$\underbrace{\underset{\text{kg/chair}}{2}\ \underset{\text{\# of chairs}}{x}}_{\text{kg for all chairs}} + \underbrace{3y}_{\text{kg for all tables}} \le \underset{\text{maximum available}}{60}$$

Metal (kg): $$4x + 4y \le 100$$

Non-negative: $$x \ge 0, \quad y \ge 0$$ (Can't have a negative number of physical objects)

4) Graph Feasible Set

$$\boxed{\begin{array}{l} 2x+3y \le 60 \\ 4x+4y \le 100 \\ x \ge 0,\ y \ge 0 \end{array}}$$



$2x+3y=60$

$(0,20)$

$(30,0)$

Test $(0,0)$: TRUE

$4x+4y=100$

$(0,25)$

$(25,0)$

Test $(0,0)$: TRUE

5) Find all Vertices (Corners)

(0, 0)

(0, 20)

(25, 0)

A:

$4x + 4y = 100$

$4y = 100 - 4x$

$y = 25 - x$

$2x + 3y = 60$

$3y = 60 - 2x$

$y = 20 - \frac{2}{3}x$

$y = y$

$25 - x = 20 - \frac{2}{3}x$

Mult. by 3: $75 - 3x = 60 - 2x$

$15 = x$

$x = 15 \rightarrow$ either equation

$x = 15 \rightarrow \quad y = 25 - x$

$y = 10$

$A = (15, 10)$

6) Objective Function

$$\text{Maximize Profit} = \underbrace{\overset{\$/\text{chair}}{14}\ \overset{\#\text{ of chairs}}{x}}_{\$\text{ for all chairs}} + \underbrace{20y}_{\$\text{ for all tables}}$$

7) Table

Recall Fundamental Theorem

Vertices	Profit = $14x + 20y$
(0,0)	0
(0,20)	400
(25,0)	350
(15,10)	410 ← maximum profit

8) Answer

The maximum profit is $410 daily, from 15 chairs and 10 tables.

Example: Astronauts have two foods: Food Alpha and Food Beta. Food Alpha has 12g of fat and 50g of carbs per serving and has a mass of 0.4kg per serving. Food Beta has 15g of fat and 20g of carbs per serving and has a mass of 0.3kg per serving. Astronauts require at least 60g of fat and 200g of carbs per day. How many servings of Food Alpha and Food Beta will minimize the total food mass per day?

1) Variables

Let x = # of servings of Food Alpha per day
" y = " Beta "

2) Chart

	Food Alpha (x)	Food Beta (y)	Required
Fat (g)	12	15	60
Carbs (g)	50	20	200
Mass (kg)	0.4	0.3	

3) Inequalities

minimum required

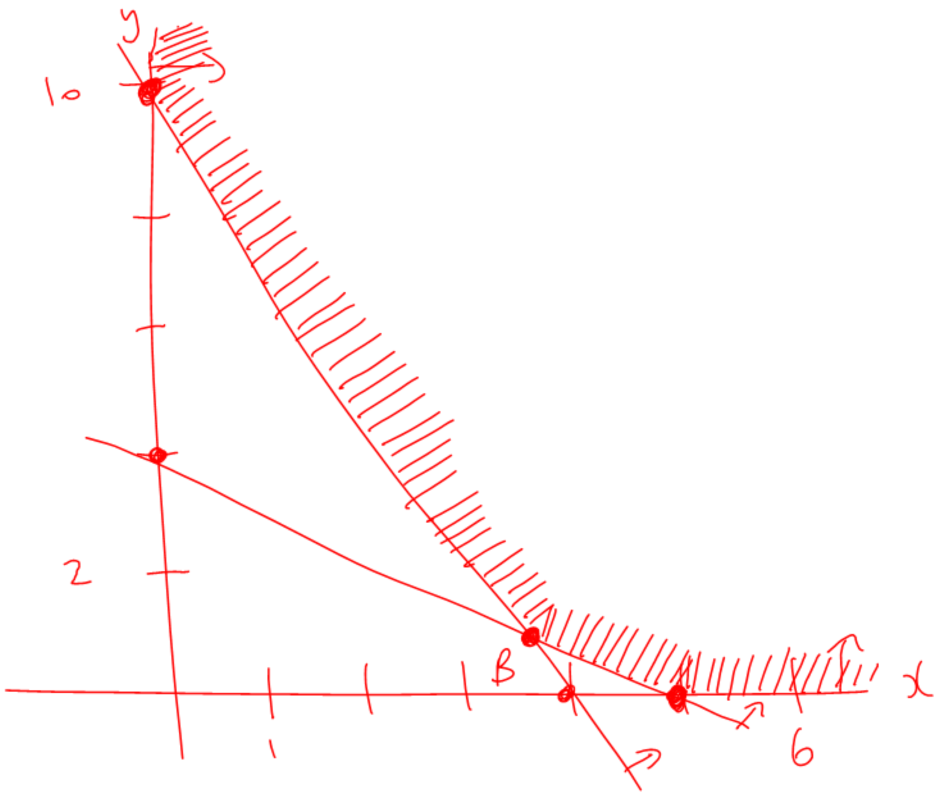
Fat (g) : $12x + 15y \geq 60$

Carbs (g) : $50x + 20y \geq 200$

Non-negative : $x \geq 0, \; y \geq 0$ (Can't have a negative number of physical objects)

4) Graph Feasible Set

$12x + 15y \geq 60$
$50x + 20y \geq 200$
$x \geq 0, \; y \geq 0$



$50x + 20y = 200$
$(4,0)$
$(0,10)$
Test $(0,0)$: FALSE

$12x + 15y = 60$
$(5,0)$
$(0,4)$
Test $(0,0)$: FALSE

5) Find all Vertices (Corners)

(0,10)

(5,0)

B: $50x + 20y = 200$

$20y = 200 - 50x$

$y = 10 - 2.5x$

$12x + 15y = 60$

$15y = 60 - 12x$

$y = 4 - \frac{12}{15}x$

$$y = y$$

$$10 - 2.5x = 4 - \frac{12}{15}x$$

Mult. by 30: $300 - 75x = 120 - 24x$

$$180 = 51x$$

$$\frac{180}{51} = x$$

$$x = \frac{180}{51} = \frac{60}{17}$$

$x = \frac{60}{17} \rightarrow$ either equation

$x = \frac{60}{17} \rightarrow$ $y = 10 - 2.5x$

$$y = 10 - 2.5\left(\frac{60}{17}\right)$$

$$y = \frac{170}{17} - \frac{150}{17}$$

$$y = \frac{20}{17}$$

$$B = \left(\frac{60}{17}, \frac{20}{17}\right)$$

6) Objective Function

Minimize $\text{Mass} = 0.4x + 0.3y$

7) Table

Recall Fundamental Theorem

Vertices	$\text{Mass} = 0.4x + 0.3y$
$(0, 10)$	3
$(5, 0)$	2
$\left(\frac{60}{17}, \frac{20}{17}\right)$	≈ 1.76 ← minimum mass

8) Answer

The minimum daily food mass is ≈ 1.76 kg, from $\frac{60}{17}$ servings of Food Alpha and $\frac{20}{17}$ " Beta.