(1)

$$\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 7 & 5 & -1 \\ 4 & 6 & 2 \\ -6 & 4 & -2 \end{bmatrix}$$

$9(7)+8(4)+7(-6)$

$$= \begin{bmatrix} 53 & 121 & -7 \\ 38 & 76 & -4 \\ 23 & 31 & -1 \end{bmatrix}$$

(2)

$$A = \begin{bmatrix} 7 & -2 \\ -3 & 8 \end{bmatrix}$$

$$D = 7(8) - (-2)(-3) = 50$$

$$A^{-1} = \frac{1}{50} \begin{bmatrix} 8 & 2 \\ 3 & 7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 8 & 2 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} -719 \\ 901 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{50} \begin{bmatrix} -3950 \\ 4150 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -79 \\ 83 \end{bmatrix}$$

$$(x, y) = (-79, 83)$$

(3) $[A \mid I]$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 \\ 5 & 3 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - 5R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -2 & -2 & 1 & 0 \\ 0 & -2 & -5 & -5 & 0 & 1 \end{array}\right]$$

$\frac{R_2}{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & -2 & -5 & -5 & 0 & 1 \end{array}\right]$$

$R_1 - R_2$, $R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & -1 & -1 & -2 & 1 \end{array}\right]$$

$\frac{R_3}{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 2 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & 2 & -1 \end{array}\right]$$

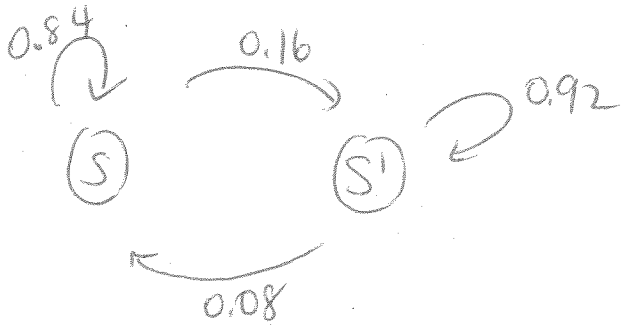
$R_2 - 2R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -5 & 2 \\ 0 & 0 & 1 & 1 & 2 & -1 \end{array}\right]$$

I $\qquad$ A^{-1}

(4) a) S : a Consumer buys SunPeak coffee

S' : a Consumer buys another brand



b)
$$P = \begin{matrix} \\ S \\ S' \end{matrix} \begin{matrix} \begin{matrix} S & S' \end{matrix} \leftarrow \text{next state} \\ \begin{bmatrix} 0.84 & 0.16 \\ 0.08 & 0.92 \end{bmatrix} \end{matrix}$$

↑ current state

c)
$$S_0 = \begin{matrix} \begin{matrix} S & S' \end{matrix} \\ \begin{bmatrix} 0.04 & 0.96 \end{bmatrix} \end{matrix}$$

$$S_1 = S_0 P$$

$$S_1 = \begin{bmatrix} 0.04 & 0.96 \end{bmatrix} \begin{bmatrix} 0.84 & 0.16 \\ 0.08 & 0.92 \end{bmatrix}$$

$$S_1 = \begin{matrix} \begin{matrix} S & S' \end{matrix} \\ \begin{bmatrix} 0.1104 & 0.8896 \end{bmatrix} \end{matrix} \leftarrow \text{next purchase}$$

$$\boxed{11.04\%}$$

(5) a) Let $S = [x \; y]$

Solve $SP = S$ together with $x + y = 1$.

$SP = S$: $[x \; y]\begin{bmatrix} 0.5 & 0.5 \\ 0.6 & 0.4 \end{bmatrix} = [x \; y]$

$[0.5x + 0.6y \quad 0.5x + 0.4y] = [x \; y]$

$0.5x + 0.6y = x$ $\qquad$ $0.5x + 0.4y = y$

$-0.5x + 0.6y = 0$ $\qquad$ $0.5x - 0.6y = 0$

$$\begin{array}{c} \quad x \quad y \\ \left[\begin{array}{cc|c} 0.5 & 0.6 & 0 \\ 0.5 & -0.6 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{l} R_2 + 0.5R_1 \\ R_3 - 0.5R_1 \end{array} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1.1 & 0.5 \\ 0 & -1.1 & -0.5 \end{array}\right]$$

$$\frac{R_2}{1.1} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & \frac{5}{11} \\ 0 & -1.1 & -0.5 \end{array}\right] \leftarrow \frac{0.5}{1.1} \times \frac{10}{10} = \frac{5}{11}$$

$$\begin{array}{l} R_1 - R_2 \\ \\ R_3 + 1.1R_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & \frac{6}{11} \\ 0 & 1 & \frac{5}{11} \\ 0 & 0 & 0 \end{array}\right] \leftarrow 1 - \frac{5}{11} = \frac{11}{11} - \frac{5}{11}$$

$x = \frac{6}{11}$

$y = \frac{5}{11}$ $\Rightarrow$ $S = \left[\frac{6}{11} \quad \frac{5}{11}\right]$

b) It looks very similar to S.

⑥ a) $r = 0.036 \quad m = 12$

$P = 20{,}000 \quad t = 4$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$A = 20{,}000\left(1 + \frac{0.036}{12}\right)^{48}$$

$$A \approx \$\,23{,}092.20$$

b) $r = 0.036 \quad m = 12$

$A = 20{,}000 \quad t = 4$

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$20{,}000 = P\left(1 + \frac{0.036}{12}\right)^{48}$$

$$\frac{20{,}000}{\left(1 + \frac{0.036}{12}\right)^{48}} = P$$

$$P \approx \$\,17{,}321.49$$

(7) a) $r = 0.048 \quad m = 12$

$FV = 25{,}000 \quad t = 6$

$i = \frac{r}{m} = 0.004 \quad n = mt = 72$

$$FV = PMT \cdot \frac{((1+i)^n - 1)}{i}$$

$$25{,}000 = PMT \cdot \frac{(1.004^{72} - 1)}{0.004}$$

$$\frac{25{,}000 \times 0.004}{(1.004^{72} - 1)} = PMT$$

$$PMT \approx \$\ 300.31$$

b) $r = 0.048 \quad m = 12$

$PV = 25{,}000 \quad t = 6$

$i = \frac{r}{m} = 0.004 \quad n = mt = 72$

$$PV = PMT \cdot \frac{(1 - (1+i)^{-n})}{i}$$

$$25{,}000 = PMT \cdot \frac{(1 - 1.004^{-72})}{0.004}$$

$$\frac{25{,}000 \times 0.004}{(1 - 1.004^{-72})} = PMT$$

$$PMT \approx \$\ 400.31$$