① Let G: The first three letters are A, B, C, in some order.

$n(S) = 6!$ or $P(6,6) = 720$

$n(G) = 3! \times 3! = 36$

($3!$: order A, B, C; $3!$: order D, E, F)

$Pr(G) = \frac{n(G)}{n(S)} = \frac{36}{720} = 0.05$

② E: at least two of the accounts choose the same avatar

E': all of the accounts choose different avatars

$n(S) = 20 \times 20 \times 20 \times 20 \times 20 \times 20 = 20^6$

(first 20: avatar for Account #1; last 20: avatar for Account #6)

$n(E') = P(20,6)$

$Pr(E') = \frac{P(20,6)}{20^6} \approx 0.44$

$Pr(E) = 1 - Pr(E') \approx 0.56$

(3) Find the probability that an employee who has been promoted recently is a college graduate.

has been promoted recently — given

$$\Pr(\text{college grad} \mid \text{recent promotion})$$

$$= \frac{\Pr(\text{college grad and recent promotion})}{\Pr(\text{recent promotion})}$$

$$= \frac{0.33}{0.55}$$

$$= 0.6$$

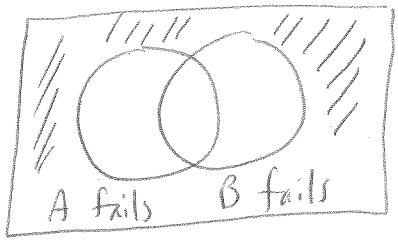
④ a) Pr (neither part fails)

= Pr (Part A doesn't fail and Part B doesn't fail)

= Pr (Part A doesn't fail) · Pr (Part B doesn't fail)
by independence

= 0.97 (0.92)

= 0.8924



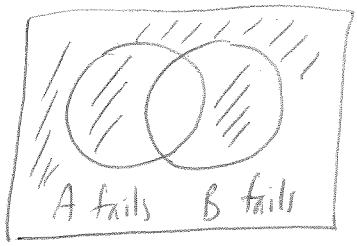
b) Pr (at least one part survives)

= 1 − Pr (both parts fail)

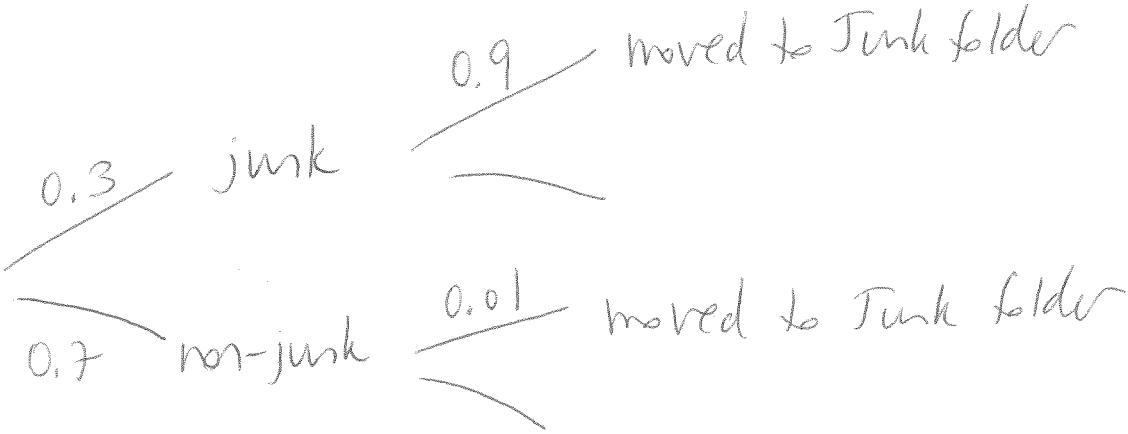
= 1 − Pr (Part A fails) · Pr (Part B fails)
by independence

= 1 − 0.03 (0.08)

= 0.9976



(5)



$$\Pr(\text{junk} \mid \text{moved to Junk folder})$$

$$= \frac{\Pr(\text{junk and moved to Junk folder})}{\Pr(\text{moved to Junk folder})}$$

$$= \frac{0.3(0.9)}{[0.3(0.9) + 0.7(0.01)]}$$

$$\approx 0.97$$

(6) a) Let X = net winnings (9)

X = (amount won) − (amount paid)

X = (twice your roll) − 5

	X	$P(X)$
roll a 1:	−3	0.1
roll a 2:	−1	0.2
roll a 3:	1	0.3
roll a 4:	3	0.4

b) $E(X)$ or $\mu = -3(0.1) - 1(0.2) + 1(0.3) + 3(0.4)$

$= 1$

We expect net winnings of \$1, on average.

(7) Binomial Experiment

n = # of trials = 8

p = P(correct guess on one question) = $\frac{1}{4}$

$q = 1 - p = \frac{3}{4}$

x = # of questions guessed correctly

$$P(x \geq 3) = 1 - P(x=0) - P(x=1) - P(x=2)$$

$$= 1 - {}_8C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^8 - {}_8C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^7 - {}_8C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6$$

$$\approx 0.32$$