# 4.4 Cont'd

## Bad Case

Any terms in $y_p$ that also appear in $y_c$ must be multiplied by the smallest power of $x$ that avoids the duplication.

Ex: Given $y_c = C_1 e^x + C_2 x e^x$

Find $y_p$ if:

a) $f(x) = 5e^x + 6e^{2x}$

$f'(x) = 5e^x + 12e^{2x}$

$y_p = Ae^x + Be^{2x}$

not bad case
no duplication

bad case
"duplication"

$y_p = Ax^2 e^x + Be^{2x}$

b) $f(x) = e^x \cos x + 1$

$f'(x) = -e^x \sin x + e^x \cos x$

$y_p = Ae^x \cos x + Be^x \sin x + C$

Not Bad Case $\quad (y_c = C_1 e^x + C_2 x e^x)$

c) $f(x) = 7xe^x$

$f'(x) = 7xe^x + 7e^x$

$f''(x) = 7xe^x + 7e^x + 7e^x$ nothing new

$$y_p = Axe^x + Be^x$$ bad case ✓

$(y_c = C_1 e^x + C_2 x e^x)$

$Axe^x \to Ax^3e^x$, $Be^x \to Bx^2e^x$

$$y_p = Ax^3e^x + Bx^2e^x$$

Also correct : $y_p = Ax^2e^x + Bx^3e^x$

Ex: Solve $y'' - 6y' + 8y = 3e^{2x}$

$y(0) = 0$ , $y'(0) = 0$

1) $y_c$

$$m^2 - 6m + 8 = 0$$

$$(m-2)(m-4) = 0$$

$$m = 2, 4$$

$$y_c = C_1 e^{2x} + C_2 e^{4x}$$

2) $y_p$

$$f(x) = 3e^{2x}$$

$$f'(x) = 6e^{2x}$$

$$\cancel{y_p = Ae^{2x}}$$ Bad Case

$$y_p = Axe^{2x}$$

3) $y_p \to DE$ (find A)

$$\begin{cases} y_p = Axe^{2x} \\ y_p' = 2Axe^{2x} + Ae^{2x} \\ y_p'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x} \end{cases}$$

$$y_p'' = 4Axe^{2x} + 4Ae^{2x}$$

$$y'' - 6y' + 8y = 3e^{2x}$$

$$4Axe^{2x} + 4Ae^{2x} - 6(2Axe^{2x} + Ae^{2x})$$
$$+ 8Axe^{2x} = 3e^{2x}$$

$$\underbrace{\boxed{4A - 12A + 8A}}_{0} xe^{2x} + \underbrace{\boxed{4A - 6A}}_{-2A} e^{2x} = 0xe^{2x} + 3e^{2x}$$

$$-2A = 3$$
$$A = \frac{-3}{2}$$
$$y_p = Axe^{2x}$$
$$y_p = \frac{-3}{2}xe^{2x}$$

4) $y = y_c + y_p$

$$\boxed{y = C_1e^{2x} + C_2e^{4x} - \frac{3}{2}xe^{2x}}$$

5) Initial Conditions

$\begin{matrix} y=0 \\ x=0 \end{matrix}$ : $\quad 0 = C_1 + C_2 \quad$ (1)

$$y' = 2C_1e^{2x} + 4C_2e^{4x} - \frac{3}{2}(2xe^{2x} + e^{2x})$$

$\begin{matrix} y'=0 \\ x=0 \end{matrix}$ : $\quad 0 = 2C_1 + 4C_2 - \frac{3}{2} \quad$ (2)

$-2\times$(1) : $\quad -2C_1 - 2C_2 = 0$

(2) : $\quad 2C_1 + 4C_2 = \frac{3}{2}$

$+$ ______________________

$$2C_2 = \frac{3}{2}$$

$C_2 = \frac{3}{4} \rightarrow ①: \quad C_1 + \frac{3}{4} = 0$

$C_1 = -\frac{3}{4}$

$$y = C_1 e^{2x} + C_2 e^{4x} - \frac{3}{2} x e^{2x}$$

$$y = -\frac{3}{4} e^{2x} + \frac{3}{4} e^{4x} - \frac{3}{2} x e^{2x}$$

Ex: Solve $y'' - y = xe^{4x}$

1) $y_c$

$$m^2 - 1 = 0$$

$$(m-1)(m+1) = 0$$

$$m = \pm 1$$

$$y_c = C_1 e^{x} + C_2 e^{-x}$$

2) $y_p$

$$f(x) = xe^{4x}$$

$$f'(x) = 4xe^{4x} + e^{4x}$$

$$f''(x) = 16xe^{4x} + 4e^{4x} + 4e^{4x}$$ nothing new

$$y_p = Axe^{4x} + Be^{4x}$$ bad case? No

3) $y_p \rightarrow DE$

$$y_p = Axe^{4x} + Be^{4x}$$

$$y_p' = 4Axe^{4x} + Ae^{4x} + 4Be^{4x}$$

$$y_p'' = 16Axe^{4x} + \underbrace{4Ae^{4x} + 4Ae^{4x}}_{8Ae^{4x}} + 16Be^{4x}$$

$$y'' - y = xe^{4x}$$

$$16Axe^{4x} + 8Ae^{4x} + 16Be^{4x} - (Axe^{4x} + Be^{4x}) = xe^{4x}$$

$$\underbrace{\boxed{16A - A}}_{15A} xe^{4x} + \underbrace{\boxed{8A + 16B - B}}_{8A+15B} e^{4x} = 1xe^{4x} + 0e^{4x}$$

$$15A = 1 \Rightarrow A = \frac{1}{15}$$

$$8A + 15B = 0 \Rightarrow \frac{8}{15} + 15B = 0$$

$$15B = \frac{-8}{15}$$

$$B = \frac{-8}{225}$$

$$y_p = Axe^{4x} + Be^{4x}$$

$$y_p = \frac{1}{15}xe^{4x} - \frac{8}{225}e^{4x}$$

4) General Solution

$$y = y_c + y_p$$ ✓

$$y = C_1e^{x} + C_2e^{-x} + \frac{1}{15}xe^{4x} - \frac{8}{225}e^{4x}$$ ✓

~~5) Initial Conditions~~