

Test 2

Thurs March 2

3.1, 4.1-4.4

4 Questions

Bring: calculator, music

Practice Problems on website

4.4 Non-homogeneous DE's with Constant Coefficients

Ex: Solve $y'' - 5y' + 6y = 5x$

1) Solve $y'' - 5y' + 6y = 0$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

$$m = 2, 3$$

$$y_c = C_1 e^{2x} + C_2 e^{3x}$$

"complementary solution"

2) Find y_p , "the particular solution"

$$f(x) = 5x$$

y_p consists of all terms in $f(x), f'(x), f''(x), \dots$
and y_p has unknown coefficients

$$f(x) = 5x$$

$$f'(x) = 5$$

$$f''(x) = 0$$

$$y_p = Ax + B$$

3) $y_p \rightarrow DE$

$$\begin{cases} y_p = Ax + B \\ y_p' = A \\ y_p'' = 0 \end{cases}$$

$$y'' - 5y' + 6y = 5x$$

$$0 - 5A + 6(Ax + B) = 5x$$

$$-5A + 6Ax + 6B = 5x$$

$$6Ax + (-5A + 6B) = 5x + 0$$

Match coefficients : $6A = 5 \Rightarrow A = \frac{5}{6}$

$$-5A + 6B = 0 \Rightarrow -\frac{25}{6} + 6B = 0$$

$$6B = \frac{25}{6}$$

$$B = \frac{25}{36}$$

$$y_p = Ax + B$$

$$y_p = \frac{5}{6}x + \frac{25}{36}$$

4) General Solution

$$y = y_c + y_p$$

$$y = C_1 e^{2x} + C_2 e^{3x} + \frac{5}{6}x + \frac{25}{36}$$

~~5) Initial Conditions~~

Quick Ex:

$f(x)$	y_p
polynomial of degree n	$A_n x^n + A_{n-1} x^{n-1} + \dots + A_1 x + A_0$
$\# e^{kx}$	$A e^{kx}$
$a \cos kx + b \sin kx$	$A \cos kx + B \sin kx$

Quick Ex:

Find y_p

a) $f(x) = 4xe^{6x}$

$$f'(x) = 24xe^{6x} + 4e^{6x}$$

$$f''(x) = 144xe^{6x} + 24e^{6x} + 24e^{6x}$$

$$y_p = Axe^{6x} + Be^{6x}$$

b) $f(x) = e^{2x} \sin 7x$

$$f'(x) = 7e^{2x} \cos 7x + 2e^{2x} \sin 7x$$

$$f''(x) = -49e^{2x} \sin 7x + 14e^{2x} \cos 7x + 14e^{2x} \cos 7x + 4e^{2x} \sin 7x$$

$$y_p = Ae^{2x} \cos 7x + Be^{2x} \sin 7x$$

Ex: Solve $y'' - 4y' + 4y = 2\sin 3x$

1) y_c

$$y'' - 4y' + 4y = 0$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$y_c = C_1 e^{2x} + C_2 x e^{2x}$$

2)

y_p

$$f(x) = 2\sin 3x$$


$$y_p = A \cos 3x + B \sin 3x$$

3) $y_p \rightarrow DE$ (Find A, B)

$$\begin{cases} y_p = A \cos 3x + B \sin 3x \\ y_p' = -3A \sin 3x + 3B \cos 3x \\ y_p'' = -9A \cos 3x - 9B \sin 3x \end{cases}$$

$$y'' - 4y' + 4y = 2 \sin 3x$$

$$\begin{aligned} -9A \cos 3x - 9B \sin 3x - 4(-3A \sin 3x + 3B \cos 3x) \\ + 4(A \cos 3x + B \sin 3x) = 2 \sin 3x \end{aligned}$$

$$\boxed{-9B + 12A + 4B} \sin 3x + \boxed{-9A - 12B + 4A} \cos 3x = 2 \sin 3x + 0 \cos 3x$$


$$\begin{cases} 12A - 5B = 2 & \textcircled{1} \\ -5A - 12B = 0 & \textcircled{2} \end{cases}$$

$$5 \times \textcircled{1} : \quad 60A - 25B = 10$$

$$12 \times \textcircled{2} : \quad -60A - 144B = 0$$

$$+ \quad \underline{\hspace{10em}}$$
$$-169B = 10$$

$$B = \frac{-10}{169} \rightarrow \textcircled{2} : \quad -5A + \frac{120}{169} = 0$$

$$-5A = \frac{-120}{169}$$

$$A = \frac{24}{169}$$

$$y_p = A \cos 3x + B \sin 3x$$

$$y_p = \frac{24}{169} \cos 3x - \frac{10}{169} \sin 3x$$

4) $y = y_c + y_p$

$$y = C_1 e^{2x} + C_2 x e^{2x} + \frac{24}{169} \cos 3x - \frac{10}{169} \sin 3x$$

5) Initial Conditions