Test Review

Methods:

Separable $\qquad f(x)\,dx = g(y)\,dy$

Linear / Bernoulli

Linear $\quad \frac{dy}{dx} + P(x)y = f(x)$

Bernoulli $\quad \frac{dy}{dx} + P(x)y = f(x)y^n$

Homogeneous - Degree

Sub $u = Ax + By + C$

Exact

Find I.F. to make it exact

(Check in this order)

Ex: Identify the method

a) $\frac{dy}{dx} = e^x y^2$ $\qquad$ Separable

b) $\frac{dy}{dx} = \frac{x+3y}{3x+y}$

Not separable

Not Linear / Bernoulli

Homogeneous - Degree

$(3x+y)\,dy = (x+3y)\,dx$

$\sqrt{x}$ has degree $\frac{1}{2}$

$\sqrt{x}\sqrt{y}$ " 1

$\sqrt{x} + 4\sqrt{y}$ " $\frac{1}{2}$

$-(x+3y)\,dx + (3x+y)\,dy = 0$ $\qquad$ Not Exact

c) $\frac{dy}{dx} = -\frac{1}{x}y + x$ $\frac{dy}{dx} + \frac{1}{x}y = x$ Linear

d) $\frac{dy}{dx} = -\frac{1}{x}y + xy^7$ Bernoulli

e) $\frac{dy}{dx} = \frac{-2e^x y}{2e^x + y}$

Not separable
Not Linear / Bernoulli
Not homogeneous-degree
Not $u = Ax + By + C$

$(2e^x + y)dy = -2e^x y\,dx$

$\underbrace{2e^x y}_{M}\,dx + \underbrace{(2e^x + y)}_{N}dy = 0$

Exact

$M_y = N_x$

f) $\frac{dy}{dx} = \frac{3 - x - y}{x + y}$

Not separable
Not Linear / Bernoulli
Not homogeneous-degree

$\boxed{\text{Sub } u = Ax + By + C \quad u = x + y}$

$$\begin{cases} u = x + y \\ \frac{du}{dx} = 1 + \frac{dy}{dx} \\ \frac{dy}{dx} = \frac{du}{dx} - 1 \end{cases}$$

$\frac{du}{dx} - 1 = \frac{3 - u}{u}$

$\frac{du}{dx} - 1 = \frac{3}{u} - 1$

$\vdots$

g) $\frac{dy}{dx} = \frac{-2y^2}{2xy + 1}$

Not separable
Not Linear / Bernoulli
Not homogeneous-degree
Not $u = Ax + By + C$

$$(2xy+1)\,dy = -2y^2\,dx$$

$$\underbrace{2y^2}_{M}dx + \underbrace{(2xy+1)}_{N}dy = 0$$

$M_y = 4y \qquad N_x = 2y \qquad$ Not exact

$\dfrac{M_y - N_x}{N}$ is a function of x? No

$$\frac{N_x - M_y}{M} = \frac{-2y}{2y^2} = -\frac{1}{y} \quad \checkmark$$

Can be made exact

<u>Ex.</u>: Solve $\dfrac{dy}{dx} = \dfrac{x+3y}{3x+y}$

Find an implicit solution.

$$(3x+y)\,dy = (x+3y)\,dx$$

Homogeneous - Degree

$$\text{Sub} \begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases}$$

$$(3x+ux)(u\,dx + x\,du) = (x+3ux)\,dx$$

$$\cancel{3ux\,dx} + 3x^2du + u^2x\,dx + ux^2du = x\,dx + \cancel{3ux\,dx}$$

$$3x^2du + ux^2du = x\,dx - u^2x\,dx$$

$$(3+u)x^2du = (1-u^2)\,x\,dx$$

$$\frac{3+u}{1-u^2}\,du = \frac{dx}{x}$$

Partial Fractions

$$\frac{3+u}{\cancel{1-u^2}\atop (1-u)(1+u)} = \frac{A}{1-u} + \frac{B}{1+u}$$

$$3+u = A(1+u) + B(1-u)$$

$u=-1$: $\quad 2 = 2B \Rightarrow B=1$

$u=1$: $\quad 4 = 2A \Rightarrow A=2$

$$\left[\frac{2}{1-u} + \frac{1}{1+u}\right]du = \frac{dx}{x}$$

$$\boxed{\int \frac{dw}{aw+b} = \frac{1}{a}\ln|aw+b| + C}$$

$$-2\ln|1-u| + \ln|1+u| = \ln|x| + C_1$$

$$-\ln|(1-u)^2| + \ln|1+u| = \ln|x| + C_1$$

$$\ln\left|\frac{1+u}{(1-u)^2}\right| = \ln|x| + C_1$$

$$\left|\frac{1+u}{(1-u)^2}\right| = \cancel{e^{\ln|x|+C_1}} \qquad e^{C_1}\cdot e^{\ln|x|}$$

$$\left|\frac{1+u}{(1-u)^2}\right| = e^{C_1}|x|$$

$$\frac{1+u}{(1-u)^2} = \pm e^{C_1} x$$

$$\frac{1+u}{(1-u)^2} = Cx$$

$$1+u = Cx(1-u)^2$$

$y = ux$

$u = \frac{y}{x}$

$$1+\frac{y}{x} = Cx\left(1-\frac{y}{x}\right)^2$$ ✓

$$x+y = Cx^2 \cancel{\left(1-\frac{y}{x}\right)^2}$$

$$\left(1-2\frac{y}{x}+\frac{y^2}{x^2}\right)$$

$$x+y = C(x^2-2xy+y^2)$$ ✓