

# Test 1

Thurs Feb 2

1.1-1.2, 2.2-2.5

5 Questions (2 Questions from Section 2.5)

Bring: calculator, music

Practice Problems on website

We'll do review on Wed Feb 1

## Section 2.5 Recap:

Ex: Solve explicitly using a substitution

$$\frac{dy}{dx} = -3 + \sqrt[3]{3x+y}$$

Can sub  $u = Ax + By + C$

$$\begin{cases} u = 3x + y \\ \frac{du}{dx} = 3 + \frac{dy}{dx} \\ \frac{dy}{dx} = \frac{du}{dx} - 3 \end{cases}$$

$$\frac{du}{dx} - 3 = -3 + \sqrt[3]{u}$$

Separable

$$\frac{du}{dx} = u^{1/3}$$

$$u^{-1/3} du = dx$$

$$\int u^{-1/3} du = \int dx$$

$$\frac{3}{2} u^{2/3} = x + C_1$$

$$\frac{3}{2} (3x+y)^{2/3} = x + C_1$$

$$(3x+y)^{2/3} = \frac{2}{3}x + C_2$$

Raise both sides to power  $3/2$ :

$$3x + y = \left(\frac{2}{3}x + C_2\right)^{3/2}$$

$$y = \left(\frac{2}{3}x + C_2\right)^{3/2} - 3x$$

Ex: Solve explicitly using a substitution

$$(x - 2y)dx + 2x dy = 0$$

↑ ↑ ↑  
all terms have degree 1

$9xy^2$  has degree 3

DE is homogeneous of degree 1

Sub  $y = ux$

$$\frac{dy}{dx} = u \frac{dx}{dx} + x \frac{du}{dx}$$

$$dy = u dx + x du$$

$$(x - 2ux)dx + 2x(u dx + x du) = 0$$

$$x dx - 2ux dx + 2ux dx + 2x^2 du = 0$$

$$x dx + 2x^2 du = 0$$

Separable

$$x dx = -2x^2 du$$

$$\frac{dx}{x} = -2 du$$

$$\int \frac{dx}{x} = \int -2 du$$

$$\ln|x| = -2u + C_1$$

$$\ln|x| = -2\left(\frac{y}{x}\right) + C_1$$

$$x \ln|x| = -2y + C_1 x$$

$$\begin{aligned} y &= ux \\ u &= \frac{y}{x} \end{aligned}$$

$$\frac{1}{2} x \ln|x| = -y + C_2 x$$

$$y = -\frac{1}{2} x \ln|x| + C_2 x$$

Ex: Solve explicitly using a substitution

$$\frac{dy}{dx} = y(y^3 - 1)$$

$$\frac{dy}{dx} = y^4 - y$$

$$\frac{dy}{dx} + y = y^4$$

Bernoulli DE  
 $\frac{dy}{dx} + p(x)y = f(x)y^n$   
 $n=4$

Sub  $y = u^{\frac{1}{1-n}}$

$$y = u^{-\frac{1}{3}}$$

$$\frac{dy}{dx} = -\frac{1}{3} u^{-\frac{4}{3}} \frac{du}{dx}$$

$$-\frac{1}{3} u^{-\frac{4}{3}} \frac{du}{dx} + u^{-\frac{1}{3}} = u^{-\frac{4}{3}}$$

Linear DE  
(Section 2.3)

Mult. by  $-3 u^{\frac{4}{3}}$ :

$$\frac{du}{dx} - 3u = -3$$

Standard Form

$$p(x) = -3$$

$$\text{I.F.} = e^{\int -3 dx}$$

$$= e^{-3x}$$

$$e^{-3x} \frac{du}{dx} - 3e^{-3x} u = -3e^{-3x}$$

Integrate w.r.t.  $x$ :

$$e^{-3x} u = e^{-3x} + C_1$$

$$y = u^{-\frac{1}{3}}$$

Raise both sides to power -3:

$$y^{-3} = u$$

$$e^{-3x} y^{-3} = e^{-3x} + C_1$$

$$\frac{1}{y^3} y^{-3} = 1 + C_1 e^{3x}$$

$$y^3 = \frac{1}{1 + C_1 e^{3x}}$$

$$y = \frac{1}{\sqrt[3]{1 + C_1 e^{3x}}}$$