Test 1

Thurs Feb 2
1.1–1.2, 2.2–2.5
5 Questions
Practice Problems on Course Webpage
We'll do review on Wed Feb 1

$\underline{\text{Ex}}$: Solve $(x + ye^{y/x})dx - xe^{y/x}dy = 0$, $y(1) = 0$

$\underline{\text{Note}}$: Ignore e^x, $\sin x$, $\ln x$ etc. when calculating degree

$\sqrt{x} = x^{1/2}$ has degree $\frac{1}{2}$

Homogeneous (of degree 1)

dy term is simpler

Sub $\begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases}$

$y/x = u$

$(x + uxe^u)dx - xe^u(u\,dx + x\,du) = 0$

$x\,dx + \underbrace{uxe^u dx - uxe^u dx}\, - x^2e^u du = 0$

$x\,dx - x^2e^u du = 0$

SEPARABLE

$x\,dx = x^2e^u du$

$\frac{1}{x}dx = e^u du$

$\int \frac{1}{x}dx = \int e^u du$

$\ln|x| = e^u + C_1$

$u = y/x$: $\ln|x| = e^{y/x} + C_1$

Sub $x=1$, $y=0$: $0 = 1 + C_1$

$C_1 = -1$

$\boxed{\ln|x| = e^{y/x} - 1}$

Bernoulli DE

$$\frac{dy}{dx} + Q(x)\,y = f(x)\,y^n$$

Like linear, but with y^n

$$\boxed{\begin{array}{l}\text{When } n \neq 0, 1 \\ \text{Sub } y = u^{\frac{1}{1-n}} \text{ to get a linear DE}\end{array}}$$

Note: When $n=0$, the DE is linear
" $n=1$ " separable

<u>Ex</u>: Solve explicitly

$$\frac{1}{y}\frac{dy}{dx} = e^x y + 1$$

$$\frac{dy}{dx} = e^x y^2 + y$$

$$\boxed{\frac{dy}{dx} - y = e^x y^2}$$

Standard Form

Bernoulli $n=2$

$$\text{Sub}\begin{cases} y = u^{\frac{1}{1-n}} = u^{\frac{1}{1-2}} = u^{-1} \\ \dfrac{dy}{dx} = -u^{-2}\dfrac{du}{dx} \end{cases}$$

$$-u^{-2}\frac{du}{dx} - u^{-1} = e^x u^{-2}$$

Mult. by $-u^2$:

$$\boxed{\frac{du}{dx} + u = -e^{x}}$$

LINEAR

$P(x) = 1$

I.F. $e^{\int P(x)dx} = e^{\int 1 dx}$

$= e^{x}$

$$e^{x}\frac{du}{dx} + e^{x}u = -e^{2x}$$

Integrate both sides:

$$e^{x}u = -\frac{1}{2}e^{2x} + C_1$$

$y = u^{-1}$

$u = y^{-1}$: $e^{x}y^{-1} = -\frac{1}{2}e^{2x} + C_1$

$$\frac{e^{x}}{C_1 - \frac{1}{2}e^{2x}} = y$$

or $y = \dfrac{2e^{x}}{C_2 - e^{2x}}$