Methods Covered so far :

Separable
Linear
Exact

Ex: Identify the method(s)

a) $\frac{dy}{dx} - x^2 + 3x^2y = 0$

$\frac{dy}{dx} = x^2 - 3x^2y$

$dy = (x^2 - 3x^2y)dx$

$dy = x^2(1-3y)\,dx$

$\frac{dy}{1-3y} = x^2\,dx$

Separable

$\frac{dy}{dx} + 3x^2y = x^2$

Linear

☆FASTEST☆

$dy + (3x^2y - x^2)dx = 0$

$(3x^2y - x^2)dx + dy = 0$

$M_y = 3x^2 \quad N_x = 0$

Not Exact

(Could find I.F.)

b) $(2xy^2 - 3)dx + (2x^2y + 4)dy = 0$

Not separable

Not linear

$M_y = 4xy = N_x$

Exact

2.5 Solutions by Substitution

Ex: $(y^2+yx)dx + x^2dy = 0$

$\uparrow \quad \uparrow \quad \uparrow$

each term has degree 2

DE is "homogeneous of degree 2"

Ex: $(3x^2y - y^3)dx - 7x^3dy = 0$

"homogeneous of degree 3"

$$\boxed{\text{Sub } \begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases} \text{ or } \begin{cases} x = vy \\ dx = v\,dy + y\,dv \end{cases} \text{ into a homogeneous DE to get a separable DE}}$$

Sub $y = ux$ when the dy term is simpler than the dx-term.

Ex: Solve explicitly

$(x^2+y^2)dx - xy\,dy = 0$

Homogeneous (of degree 2)

dy term is simpler

Sub $\begin{cases} y = ux \\ dy = u\,dx + x\,du \end{cases}$

$$(x^2 + u^2x^2)dx \quad - x(ux)(udx + xdu) = 0$$

$$x^2dx + u^2x^2dx \quad - ux^2(udx + xdu) = 0$$

$$x^2dx + \underbrace{u^2x^2dx - u^2x^2dx} \quad - ux^3du = 0$$

$$x^2dx - ux^3du = 0 \qquad \text{SEPARABLE}$$

$$x^2dx = ux^3du$$

$$\frac{1}{x}dx = udu$$

$$\int \frac{1}{x}dx = \int udu$$

$$\ln|x| = \frac{u^2}{2} + C_1$$

$y = ux$
$u = \frac{y}{x}$:

$$\ln|x| = \frac{1}{2}\frac{y^2}{x^2} + C_1$$

$$2\ln|x| = \frac{y^2}{x^2} + C_2$$

$$2\ln|x| + C_3 = \frac{y^2}{x^2}$$

$$\pm\sqrt{2\ln|x| + C_3} = \frac{y}{x}$$

$$y = \pm x\sqrt{2\ln|x| + C_3}$$