Do Sugg HW for Sections 1.1, 1.2, 2.2

2.2 Cont'd

Ex: Solve $\frac{dy}{dx} = y^2 - 9$

⋮

$$\ln|y-3| - \ln|y+3| = 6x + C_2$$

$$\ln\frac{|y-3|}{|y+3|} = 6x + C_2$$

$e^{LS} = e^{RS}$:

$$\frac{|y-3|}{|y+3|} = e^{6x+C_2}$$

$$\frac{y-3}{y+3} = \pm e^{C_2} e^{6x}$$

$$\frac{y-3}{y+3} = Ce^{6x}$$

$$y-3 = Ce^{6x}(y+3)$$

$$y-3 = Ce^{6x}y + 3Ce^{6x}$$

$$y - Ce^{6x}y = 3 + 3Ce^{6x}$$

$$y(1-Ce^{6x}) = 3(1+Ce^{6x})$$

$$y = \frac{3(1+Ce^{6x})}{1-Ce^{6x}}$$

$y-3 = Ce^{6x}(y+3)$ IMPLICIT SOLUTION

standard form for implicit solution:

Get rid of ln, absolute values, fractions

$y = \frac{3(1+Ce^{6x})}{1-Ce^{6x}}$ EXPLICIT SOLUTION

Ex: Find an implicit solution

$$\frac{dy}{dx} = \frac{(x-1)(y+3)}{(x+4)(y-2)}$$

$$dy = \frac{(x-1)(y+3)}{(x+4)(y-2)}\,dx$$

$$\frac{(y-2)\,dy}{y+3} = \frac{(x-1)\,dx}{x+4}$$

$$\int \frac{y-2}{y+3}\,dy = \int \frac{x-1}{x+4}\,dx$$

Long Division

$$
\boxed{
\begin{array}{r}
1 \\
y+3\,\overline{)\,y-2} \\
-(y+3) \\
\hline
-5
\end{array}
\qquad
\frac{y-2}{y+3} = 1 - \frac{5}{y+3}
}
$$

$$
\boxed{
\begin{array}{r}
1 \\
x+4\,\overline{)\,x-1} \\
-(x+4) \\
\hline
-5
\end{array}
\qquad
\frac{x-1}{x+4} = 1 - \frac{5}{x+4}
}
$$

$$\int\left(1-\frac{5}{y+3}\right)dy = \int\left(1-\frac{5}{x+4}\right)dx$$

$$y - 5\ln|y+3| = x - 5\ln|x+4| + C_1$$

$$y + \ln|y+3|^{-5} = x + \ln|x+4|^{-5} + C_1$$

$e^{LS} = e^{RS}$:

$$e^{y+\ln|y+3|^{-5}} = e^{x+\ln|x+4|^{-5}+C_1}$$

$$e^{y} \cdot e^{\ln|y+3|^{-5}} = e^{c_1} \cdot e^{x} \cdot e^{\ln|x+4|^{-5}}$$

$$e^{y} |y+3|^{-5} = e^{c_1} e^{x} |x+4|^{-5}$$

$$e^{y} |x+4|^{5} = e^{c_1} e^{x} |y+3|^{5}$$

$$e^{y} (x+4)^{5} = \pm e^{c_1} e^{x} (y+3)^{5}$$

$$e^{y} (x+4)^{5} = C e^{x} (y+3)^{5}$$

Ex: Find an Implicit solution

$$\left(e^{3y} - y e^{y}\right) \frac{dy}{dx} = e^{2y} \cos 5x \quad , \qquad y(0) = 7$$

$$\left(e^{y} - y e^{-y}\right) \frac{dy}{dx} = \cos 5x$$

$$\left(e^{y} - y e^{-y}\right) dy = \cos 5x \, dx$$

Integration by Parts

	D	I
$\oplus$	$-y$	e^{-y}
$\ominus$	-1	$-e^{-y}$
		e^{-y}

$$\int -y e^{-y} dy = y e^{-y} + e^{-y} + \text{Constant}$$

$$\int \left(e^{y} - y e^{-y}\right) dy = \int \cos 5x \, dx$$

$$e^{y} + y e^{-y} + e^{-y} = \frac{\sin 5x}{5} + C_1$$

$$5\left[e^{y} + (y+1) e^{-y}\right] = \sin 5x + \cancel{5C_1} \; C_2$$

$x=0$
$y=7$: $\quad 5\left[e^{7}+8e^{-7}\right] = C_2$

$$5\left[e^{y}+(y+1)e^{-y}\right] = \sin 5x + 5\left[e^{7}+8e^{-7}\right]$$

Negative exponents are ok if the solution has 3 or more terms.

Ex: Find an explicit solution. Interval of solution?

$$y' - ty^3 = 0, \qquad y(0)=2$$

$$\frac{dy}{dt} - ty^3 = 0$$

$$\frac{dy}{dt} = ty^3$$

$$y^{-3}\,dy = t\,dt$$

$$\int y^{-3}\,dy = \int t\,dt$$

$$-\frac{1}{2}y^{-2} = \frac{t^2}{2} + C_1$$

$y=2$
$t=0$: $\quad -\frac{1}{2}\left(\frac{1}{4}\right) = C_1$

$$C_1 = -\frac{1}{8}$$

$$-\frac{1}{2}y^{-2} = \frac{t^2}{2} - \frac{1}{8}$$

$$4y^{-2} = -4t^2 + 1$$

$$\frac{4}{y^2} = 1 - 4t^2$$

$$\frac{4}{1-4t^2} = y^2$$

$$y = \pm\sqrt{\frac{4}{1-4t^2}}$$

$$y = \pm\frac{2}{\sqrt{1-4t^2}}$$

Choose $\oplus$ because $y(0)=2$

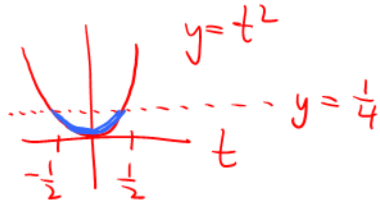
$$y = \frac{2}{\sqrt{1-4t^2}}$$

Interval: $1-4t^2 > 0$

$$1 > 4t^2$$

$$\frac{1}{4} > t^2$$

$$t^2 < \frac{1}{4}$$



$$|t| < \frac{1}{2}$$

or $-\frac{1}{2} < t < \frac{1}{2}$