Practice Problem #30

⋮

$$y(t) = \mathcal{L}^{-1}\left\{ \frac{\frac{1}{5}s + \frac{1}{10}}{s^2+1} + \frac{-\frac{1}{5}s - \frac{1}{2}}{s^2+2s+5} + \frac{2s+3}{s^2+2s+5} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{\frac{1}{5}s + \frac{1}{10}}{s^2+1} + \frac{\frac{9}{5}s + \frac{5}{2}}{s^2+2s+5} \right\}$$

$$s^2+2s+5 = (s+1)^2 + ?$$
$$= (s+1)^2 + \cancel{4}\, 2^2$$

$$\frac{9}{5}s + \frac{5}{2} = ?(s+1) + ?$$
$$= \frac{9}{5}(s+1) + ?$$

$$\frac{5}{2} - \frac{9}{5} = \frac{7}{10}$$

$$= \frac{9}{5}(s+1) + \frac{7}{10}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{5}\frac{s}{s^2+1} + \frac{1}{10}\frac{1}{s^2+1} + \frac{\frac{9}{5}(s+1) + \frac{7}{10}}{(s+1)^2+2^2} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{5}\frac{s}{s^2+1} + \frac{1}{10}\frac{1}{s^2+1} + \frac{9}{5}\frac{s+1}{(s+1)^2+2^2} + \frac{7}{\cancel{10}\,20}\frac{\cancel{1}\,2}{(s+1)^2+2^2} \right\}$$

$$= \frac{1}{5}\cos t + \frac{1}{10}\sin t + \frac{9}{5}e^{-t}\cos 2t + \frac{7}{20}e^{-t}\sin 2t$$

Sugg HW 8.2 #4

Solve $\vec{x}\,' = \begin{bmatrix} 3 & -1 \\ 9 & -3 \end{bmatrix}\vec{x}$

$$\begin{vmatrix} 3-\lambda & -1 \\ 9 & -3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-3-\lambda) + 9 = 0$$

$$-9 + \lambda^2 + 9 = 0$$

$$\lambda^2 = 0$$

$$\lambda = 0, 0$$

$\lambda = 0$:

$$[A - \lambda I \mid \vec{0}]$$

$$[A \mid \vec{0}]$$

$$\left[\begin{array}{cc|c} 3 & -1 & 0 \\ 9 & -3 & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cc|c} 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$$x_2 = a$$

$$x_1 - \frac{1}{3}x_2 = 0 \implies x_1 = \frac{1}{3}a$$

$$\vec{x} = \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix} a$$

$$a = 3 \implies \vec{K} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Not enough eigenvectors.

$$[A - \lambda I]\vec{P} = \vec{K}$$

$\lambda = 0$: $$[A]\vec{P} = \vec{K}$$

$$\left[\begin{array}{cc|c} 3 & -1 & 1 \\ 9 & -3 & 3 \end{array}\right]$$

$$\rightsquigarrow \begin{array}{cc} p_1 & p_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$$p_2 = a$$

$$p_1 - \frac{1}{3}p_2 = \frac{1}{3} \implies p_1 = \frac{1}{3} + \frac{1}{3}a$$

$$\vec{P} = \begin{bmatrix} \frac{1}{3} + \frac{1}{3}a \\ a \end{bmatrix}$$

Choose any nonzero $\vec{P}$: $a = 2 \implies \vec{P} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

or $\begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix}$ etc.

$$\vec{x} = c_1 \vec{k} e^{\lambda t} + c_2 (\vec{k} t + \vec{p}) e^{\lambda t}$$

$$\vec{x} = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \left(\begin{bmatrix} 1 \\ 3 \end{bmatrix} t + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right)$$

Sugg HW 8.2 # 7

Solve $\vec{x}' = \begin{bmatrix} 6 & -1 \\ 5 & 2 \end{bmatrix} \vec{x}$

$$\begin{vmatrix} 6-\lambda & -1 \\ 5 & 2-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)(2-\lambda) + 5 = 0$$

$$\lambda^2 - 8\lambda + 17 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64-68}}{2}$$

$$\lambda = \frac{8 \pm \cancel{\sqrt{-4}}}{2} \quad \sqrt{4}\sqrt{-1}$$

$$\lambda = \frac{8 \pm 2i}{2}$$

$$\lambda = 4 \pm i$$

$\lambda = 4 + i$:

$$[\, A - (4+i) I \mid \vec{0} \,]$$

$$\left[\begin{array}{cc|c} 2-i & -1 & 0 \\ 5 & -2-i & 0 \end{array}\right]$$

$\frac{R_2}{5}$ then $R_2 \leftrightarrow R_1$

$$\left[\begin{array}{cc|c} 1 & \frac{-2-i}{5} & 0 \\ 2-i & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{-2-i}{5} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

A zero row is guaranteed
(know there are nontrivial solutions).

$\uparrow$

$$x_2 = a$$

$$x_1 + \frac{-2-i}{5} x_2 = 0$$

$$\Rightarrow \quad x_1 = \frac{2+i}{5} a$$

$$\vec{x} = \begin{bmatrix} \frac{2+i}{5} \\ 1 \end{bmatrix} a$$

$$a = 5 \quad \Rightarrow \quad \vec{K} = \begin{bmatrix} 2+i \\ 5 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}_{\vec{B}_1} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\vec{B}_2} i$$

$$\vec{X} = C_1 e^{\alpha t}\left[\vec{B}_1 \cos\beta t - \vec{B}_2 \sin\beta t\right]$$
$$+ C_2 e^{\alpha t}\left[\vec{B}_1 \sin\beta t + \vec{B}_2 \cos\beta t\right]$$

$\lambda = 4 + i$
$\alpha = 4, \beta = 1$

$$\vec{X} = C_1 e^{4t}\left[\begin{bmatrix} 2 \\ 5 \end{bmatrix}\cos t - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\sin t\right]$$
$$+ C_2 e^{4t}\left[\begin{bmatrix} 2 \\ 5 \end{bmatrix}\sin t + \begin{bmatrix} 1 \\ 0 \end{bmatrix}\cos t\right]$$