Test Average: 73%

Formula Sheet has been updated.

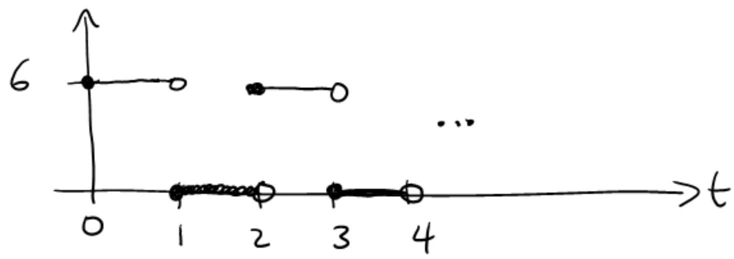
Removed: 3 Trig Sub
$\cos 2x$
$\tan 2x$
$\tan(A \pm B)$

Assignment Coverage 7.1-7.5
Handed Out: Tues Apr 4^{th}
Due: Tues Apr 11^{th}, 8:30am

Omitting Section 8.3

7.4 Cont'd

Periodic Functions



$f(t)$ is periodic with period $T = 2$

FACT

If $f(t)$ has period T then

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$

Ex: Find $\mathcal{L}\{f(t)\}$ for $f(t)$ shown above. Simplify the answer.

Periodic with $T=2$

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-2s}} \int_0^2 e^{-st} f(t)\, dt$$

$$= \frac{1}{1-e^{-2s}} \left[\int_0^1 6e^{-st}\,dt + \cancel{\int_1^2} 0\,dt \right]$$

$$= \frac{1}{1-e^{-2s}} \left[\frac{6}{-s} e^{-st} \right]_{t=0}^{t=1}$$

$$= \frac{1}{1-e^{-2s}} \left[-\frac{6}{s} e^{-s} + \frac{6}{s} \right]$$

$$= \frac{6}{s} \frac{1}{1-e^{-2s}} \left(1 - e^{-s} \right)$$

$$= \frac{6\left(1-e^{-s}\right)}{s\left(1-e^{-s}\right)\left(1+e^{-s}\right)}$$

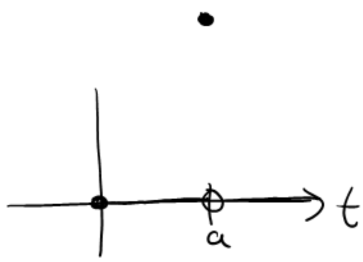
$$= \frac{6}{s\left(1+e^{-s}\right)}$$

7.5 The Dirac Delta

Let $a \geq 0$

The Dirac Delta is:

$$\delta(t-a) = \begin{cases} \infty, & t = a \\ 0, & t \neq a \end{cases}$$



$\delta(t-a)$ is a "generalized function"
or "distribution".

Models: impulses in signal processing
point charges
point masses

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}$$

Ex: 1 kg mass at rest at equilibrium position.
Mass is struck at $t=\pi$ seconds,
corresponding to an external force of $f(t) = \delta(t-\pi)$.
No damping.
Spring constant = 1 N/m.
Find $x(t)$ using $\mathcal{L}$.

$$mx'' + \beta x' + kx = f(t)$$

$m = 1\,kg$ $\quad$ $\beta = 0$ $\quad$ $k = 1\ N/m$ $\quad$ $\delta(t-\pi)$

$$x'' + x = \delta(t-\pi)$$

$$\boxed{\begin{aligned} x'(0) &= 0 \\ x(0) &= 0 \end{aligned}}$$

1) Apply $\mathcal{L}$

$$s^2 X(s) - s\,x(0) - x'(0) + X(s) = e^{-\pi s}$$

$$s^2 X(s) + X(s) = e^{-\pi s}$$

2) Solve for $X(s)$

$$(s^2+1)\,X(s) = e^{-\pi s}$$

$$X(s) = \frac{1}{s^2+1}\,e^{-\pi s}$$

3) Apply $\mathcal{L}^{-1}$

$$x(t) = f(t-\pi)\,\mathcal{U}(t-\pi)$$

$$\boxed{\begin{aligned} F(s) &= \frac{1}{s^2+1} \\ f(t) &= \sin t \\ f(t-\pi) &= \sin(t-\pi) \end{aligned}}$$

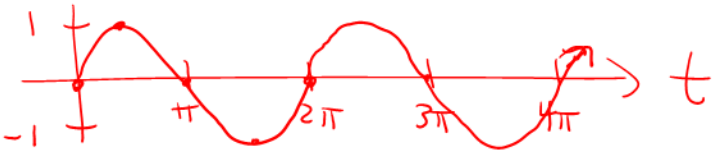
$$x(t) = \sin(t-\pi)\,\mathcal{U}(t-\pi)$$

Note: $\sin(t-\pi) = \sin t \cos \pi - \cos t \sin \pi$
$= -\sin t$

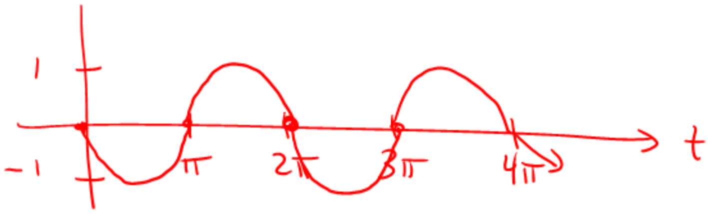
Alternatively: $x(t) = -\sin t \, \mathcal{U}(t-\pi)$

Follow-Up:

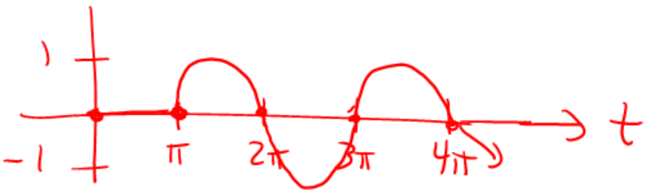
$f(t) = \sin t$



$f(t) = -\sin t$



$f(t) = -\sin t \, \mathcal{U}(t-\pi)$



Could be written $f(t) = \begin{cases} 0, & t < \pi \\ -\sin t, & t \geq \pi \end{cases}$