Test 3 moving to Tues March 28

Do Sugg HW for Section 1.1

List of problems on website

Problems " D2L

1.1 Cont'd

Ex: Show that $x^2+y^2=25$ is a solution to $\frac{dy}{dx} = -\frac{x}{y}$

Find $\frac{dy}{dx}$ by implicit differentiation

$$x^2+y^2=25$$
$$2x+2y\frac{dy}{dx}=0$$
$$2y\frac{dy}{dx}=-2x$$
$$\frac{dy}{dx}=-\frac{2x}{2y}$$
$$\frac{dy}{dx}=-\frac{x}{y}$$

LS of DE = RS of DE ✓

Notation for Derivatives:

$y', y'', y''', y^{(4)}, y^{(5)}$ etc.

$\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ etc.

Order of a DE:
order of the highest derivative

Ex: a) $y'' + 5(y')^3 + y = 0$ 2nd order DE

b) $x^6 y''' - y = 0$ 3rd order DE

c) $\frac{dy}{dx} = 25 + y^2$ 1st order DE

Normal form of a DE:
Isolate the highest derivative

Ex: $5x y'' + y = x$

$5x y'' = x - y$

$y'' = \frac{x - y}{5x}$ normal form

Linear DE: $y, y', y'', \ldots$ all to 1st power
And Coefficients don't involve y.

Ex: a) $3y'' + 2y = 0$ Linear DE

or $3y'' + 0y' + 2y = 0$

b) $8y' + e^{x} y = 0$ Linear DE

$$\boxed{\begin{array}{ll} 8y' + e^{y} y = 0 & \text{Non-linear DE} \\ 8y' + y = e^{y} & \text{Non-linear DE} \end{array}}$$

c) $(\sin x) y' + 10y = 0$ Linear DE

$(\sin x) y' + 10y = e^{x}$ "

d) $(\sin y) y' + 10y = 0$ Non-linear DE

e) $y^2 = y''$ Non-linear DE

Ex: Confirm that $y = C_1 \cos x + C_2 \sin x$
solves $y'' + y = 0$

Note: C_1, C_2 : any real #

C_1, C_2 are called parameters

$$\begin{cases} y = C_1 \cos x + C_2 \sin x \\ y' = -C_1 \sin x + C_2 \cos x \\ y'' = -C_1 \cos x - C_2 \sin x \end{cases}$$

$$\begin{aligned} \text{LS of DE} &= y'' + y \\ &= -C_1 \cos x - C_2 \sin x + C_1 \cos x + C_2 \sin x \\ &= 0 \\ &= \text{RS of DE} \end{aligned}$$

Terminology: $y = C_1 \cos x + C_2 \sin x$ is a "2-parameter family of solutions"

Some particular solutions:

$y = 0$

$y = 8 \cos x$

$y = -4 \sin x$

$y = \sqrt{2} \cos x - \pi \sin x$

etc.

Ex: Confirm that $x = Ct^4$ is a solution to $t\frac{dx}{dt} - 4x = 0$

Note: C is a parameter

$$\begin{cases} x = Ct^4 \\ \frac{dx}{dt} = 4Ct^3 \end{cases}$$

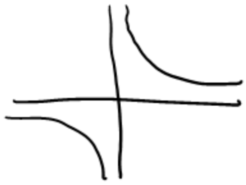
$$\begin{aligned} \text{LS of DE} &= t\frac{dx}{dt} - 4x \\ &= t(4Ct^3) - 4(Ct^4) \end{aligned}$$

$= 4Ct^4 - 4Ct^4$
$= 0$
$=$ RS of DE ✓

Terminology: $x = Ct^4$ is a "1-parameter family of solutions"

Interval of Solution:
Largest interval of x-values on which the solution y is continuous.
There may be several possible intervals.

Ex: Solution $y = \frac{1}{x}$



Continuous on $-\infty < x < 0$ and $0 < x < \infty$
Interval of Solution: $-\infty < x < 0$ or $0 < x < \infty$

Ex: Solution $y = \frac{1}{x(x+5)}$
$x \neq 0, -5$
Possible intervals: $-\infty < x < -5$, $-5 < x < 0$, $0 < x < \infty$
Interval of solution: $0 < x < \infty$

1.2 Initial Value Problems (IVP)

Ex: $y' = y$ has solution $y = Ce^x$.

Solve $\left.\begin{array}{l} y' = y \\ y(0) = 3 \end{array}\right\}$ IVP

initial condition

$$y = Ce^x$$

$y=3$
$x=0$:
$$3 = C\cancel{e^0}$$

$$\boxed{y = 3e^x}$$