3.3 Cont'd

Geometric Series: A sum in which the next term is the previous term multiplied by r

r = common ratio

$$7 + 14 + 28 + \dots$$

Fact

For a geometric series

$$S_k = \frac{a_m (1 - r^k)}{1 - r}$$

S_k = sum of first k terms

a_m = first term

r = common ratio

k = # of terms being summed

Ex: Consider $2 + 10 + 50 + \dots$

Find the sum of the first 12 terms.

Geometric Series $r = 5$ $a_1 = 2$

$$S_k = \frac{a_m (1 - r^k)}{1 - r}$$

$k = 12$: $m = 1$

$$S_{12} = \frac{a_1 (1 - r^{12})}{1 - r}$$

$$= \frac{2 (1 - 5^{12})}{(1 - 5)}$$

$= 122070312$

Fact

If $-1 < r < 1$

then $r^k \to 0$ as $k \to \infty$

Quick ex:

$$0.9^{100} \approx 0$$

$$(-0.8)^{200} \approx 0$$

Fact

If $-1 < r < 1$ then $S_\infty = \dfrac{a_m}{1-r}$

If $r \geq 1$ or $r \leq -1$ then S_∞ is undefined.

Why?

$$S_k = \frac{a_m\left(1-r^k\right)}{1-r}$$

if $-1 < r < 1$ ← this goes to zero as $k \to \infty$

Ex: Calculate $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$

Geometric Series $r=\frac{1}{2}$ $a_1=\frac{1}{2}$

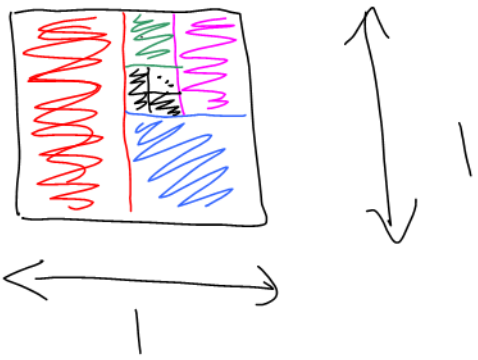
$$S_\infty = \frac{a_1}{1-r} \quad \text{if } -1<r<1$$

$$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}$$

$$= \frac{1}{2} \times \frac{2}{1}$$

$$= 1$$

Visualization:



$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots = 1$$

Ex: Calculate $24-16+\frac{32}{3}-\ldots$

Geometric Series $r=\frac{-16}{24}=\frac{-2}{3}$

$$S_\infty=\frac{a_m}{1-r} \quad \text{if } -1<r<1$$

$$=\frac{24}{\left(1+\frac{2}{3}\right)}$$

$$=\frac{24}{\left(\frac{5}{3}\right)}$$

$$=24\times\frac{3}{5}$$

$$=\frac{72}{5} \text{ or } 14.4$$

Ex: Calculate $12+18+27+\ldots$

Geometric Series $r=\frac{18}{12}=\frac{3}{2}$

S_∞ is undefined
because r is not
in the range $-1 < r < 1$.