FINAL EXAM

Mon April 22
8:30 am (three hours long)
CHW 351

Quiz Tues Feb 27 Section 3.1

3.2 Arithmetic Sequences and Series Cont'd

Arithmetic Sequence: $\underbrace{9, 13}_{+4}, \underbrace{\ , 17}_{+4}, \ldots$

Arithmetic Series: $\underbrace{9 + 13}_{+4} + \underbrace{\ 17}_{+4} + \ldots$

Recall S_k is the sum of the first k terms.

FACT
For an arithmetic series:

$$S_k = \frac{k}{2}(a_m + a_n)$$

$$S_k = \frac{k}{2}\left[2a_m + (n-m)d\right]$$

m = smaller index
n = larger index
k = # of terms
$= n - m + 1$

Ex: Find the sum of the first 50 terms of $2+5+8+\dots$

Arithmetic series $d=3$, $a_1=2$

Don't know a_{50}, use 2^{nd} formula.

$$S_k = \frac{k}{2}\left[2a_m + (n-m)d\right]$$

$m=1$

$n=50$

$k=50$

$$\therefore \; S_{50} = \frac{50}{2}\left[2a_1 + 49d\right]$$

$$= 25\left[2(2)+49(3)\right]$$

$$= 3775$$

Ex: Evaluate $\sum_{j=4}^{50}(6j-3)$

$$= \underset{a_m}{21} + 27 + \dots + \underset{a_n}{297}$$

Use $S_k = \frac{k}{2}(a_m + a_n)$

$$k = \# \text{ terms}$$
$$= n - m + 1$$
$$= 50 - 4 + 1$$
$$= 47$$

$$S_k = \frac{k}{2}(a_m + a_n)$$
$$= \frac{47}{2}(21 + 297)$$
$$= 7473$$

3.3 Geometric Sequences and Series

A geometric sequence is a sequence in which the next term is the previous term times a constant.

The constant is called the common ratio, written r.

Ex: Find r in the geometric sequences below:

a) $7, 14, 28, \ldots$

$7 \xrightarrow{\times 2} 14 \xrightarrow{\times 2} 28$

$r = 2$

$r = \frac{14}{7} = 2$

b) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$\frac{1}{2} \xrightarrow{\times \frac{1}{2}} \frac{1}{4} \xrightarrow{\times \frac{1}{2}} \frac{1}{8} \longrightarrow \frac{1}{16}$

$r = \frac{1}{2}$

$r = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{4} \times \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$

c) $24, -8, \frac{8}{3}, -\frac{8}{9}, \ldots$

$r = -\frac{1}{3}$

$r = \frac{-8}{24} = -\frac{1}{3}$

Recursive Formula for Infinite Geometric Sequences

$$\begin{cases} a_m = \text{<insert first term here>} \\ a_n = r\, a_{n-1} \quad \text{for } n \geq m+1 \end{cases}$$

Ex: Give a recursive formula for $100, 20, 4, \frac{4}{5}, \ldots$

Geometric Sequence

$a_1 = 100$

$r = \frac{20}{100} = \frac{1}{5}$

$$\begin{cases} a_1 = 100 \\ a_n = \frac{1}{5} a_{n-1}, \quad n \geq 2 \end{cases}$$

General Formula for Infinite Geometric Sequences

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

<u>Ex</u>: Find a general formula for $80, -160, 320, \ldots$

Geometric $\quad a_1 = 80 \quad r = \frac{-160}{80} = \frac{-2}{1} = -2$

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

$m = 1$: $\quad a_n = a_1 r^{n-1} \quad \text{for } n \geq 1$

$$a_n = 80(-2)^{n-1} \quad \text{for } n \geq 1$$

<u>Ex</u>: Consider $5, 15, 45, \ldots$
Find the twelfth term.

Geometric Sequence

$a_1 = 5 \qquad r = \frac{15}{5} = 3$

$$a_n = a_m r^{n-m} \quad \text{for } n \geq m$$

$m=1$: $\quad a_n = a_1 r^{n-1} \quad \text{for } n \geq 1$

$$a_n = 5 \cdot 3^{n-1} \quad \text{for } n \geq 1$$

$n = 12$: $\quad a_{12} = 5 \cdot 3^{11}$

$$= 885735$$

Geometric Series:

$$2 + 10 + 50 + 250 + \ldots$$