3.2 Arithmetic Sequences and Series

An **arithmetic sequence** is a sequence in which the next term is the previous term plus a constant, called the **common difference** d.

Ex: $5, -5, -15, -25, \ldots$

$-10 \quad -10 \quad -10$

$d = -10$

Formally: $d = a_2 - a_1$

$= -5 - 5$

$= -10$

Recursive formula for an Infinite Arithmetic Sequence

$$\begin{cases} a_m = \text{<insert first term here>} \\ a_n = a_{n-1} + d, & n \geq m+1 \end{cases}$$

Ex: Write the recursive formula for $0.4, 0.5, 0.6, \ldots$

arithmetic sequence

$d = a_2 - a_1$

$= 0.5 - 0.4$

$= 0.1$

$$\begin{cases} a_1 = 0.4 \\ a_n = a_{n-1} + 0.1, & n \geq 2 \end{cases}$$

General formula for an Infinite Arithmetic Sequence

$$a_n = a_m + (n-m)d \text{ for } n \geq m$$

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Ex: Find the simplified general formula for $5, -5, -15, \ldots$

-10 -10

arithmetic sequence

$d = a_2 - a_1$

$= -5 - 5$

$= -10$

$a_n = a_m + (n-m)d$ for $n \geq m$

Sub $m=1$: $a_n = a_1 + (n-1)d$ for $n \geq 1$

Sub $a_1 = 5$, $d = -10$: $a_n = 5 + (n-1)(-10)$ for $n \geq 1$

$a_n = 5 - 10n + 10$

$a_n = 15 - 10n$

Ex: An arithmetic sequence has $a_1 = 2$ and $a_{50} = 394$. Find d.

$a_n = a_m + (n-m)d$

n = larger index
m = smaller index

Sub $m = 1$, $n = 50$: $a_{50} = a_1 + 49d$

$394 = 2 + 49d$

$392 = 49d$

$8 = d$

$d = 8$

Ex: An arithmetic sequence has $a_1 = 3$ and $d = 7$.

a) Find a_{21}

$a_n = a_m + (n-m)d$

$n = 21$

Sub $m = 1$: $a_{21} = a_1 + 20(7)$

$d = 7$

sub $a_1 = 3$: $a_{21} = 3 + 20(7)$

$= 143$

b) Which term equals 717?

$$a_n = a_m + (n-m)d$$

Sub $m=1$, $d=7$: $a_n = a_1 + (n-1)(7)$

Sub $a_1=3$, $a_n=717$:

$$717 = 3 + (n-1)(7)$$
$$714 = (n-1)(7)$$
$$102 = n-1$$
$$103 = n$$

a_{103} equals 717.