Quiz Tues Feb 13 Section 2.5

Test Wed Feb 14
2.3 – 2.8
7 Questions
Practice Problems on website

Quiz Tues Feb 27 Section 3.1

3.1 Sequences and Series Cont'd

3 ways to define a sequence:

LIST $3, 5, 7, \ldots, 13$

GENERAL FORMULA $a_n = 2n+1$, for $1 \le n \le 6$

RECURSIVE FORMULA

$$\begin{cases} a_1 = 3 \\ a_n = 2 + a_{n-1} \text{ for } 2 \le n \le 6 \end{cases}$$

a_n: next term

a_{n-1}: previous term

Ex: Find the first four terms.

a) $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 \end{cases}$ for $n \geq 2$

$a_1 = 2$, $a_2 = a_1 + 3 = 5$,

$a_3 = a_2 + 3 = 8$,

$a_4 = a_3 + 3 = 11$

b) $\begin{cases} a_1 = 1, \quad a_2 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$ for $n \geq 3$

$a_1 = 1$

$a_2 = 1$

$a_3 = a_2 + a_1 = 2$

$a_4 = a_3 + a_2 = 3$

Factorials

$n! = n(n-1)(n-2)(n-3)\cdots 1$

"n factorial"

$4! = 4(3)(2)(1) = 24$

$3! = 3(2)(1) = 6$

$2! = 2(1) = 2$

$1! = 1$

$(0! = 1$ by definition$)$

Ex: Write a recursive formula for $1, 2, 6, 24, 120, 720, \ldots$

$$\begin{cases} a_1 = 1 \\ a_n = n\, a_{n-1}, & n \geq 2 \end{cases}$$

(TRICKY)

Series: sum of numbers

(Sequence: list of numbers)

$5+15+25+\ldots+105$ is a finite series

$5+15+25+\ldots$ is an infinite series

S_k = sum of first k terms of a series

S_∞ = sum of all terms of an infinite series

Ex: Consider $16+20+24+\dots$
Find S_3 and S_5.

$S_3 = 16+20+24 = 60$

$S_5 = 16+20+24+28+32 = 120$

Note: The series $a_1+a_2+a_3+\dots$
has $S_3 = a_1+a_2+a_3$

The series $a_0+a_1+a_2+\dots$
has $S_3 = a_0+a_1+a_2$

Sigma Notation

$$\sum_{n=1}^{4}(3n+1) = \underset{(n=1)}{4} + 7 + 10 + \underset{(n=4)}{13} = 34$$

"Sum from $n=1$ to 4 of $3n+1$"

Ex: Evaluate

a) $\sum_{i=0}^{2} 3^i = \underset{(i=0)}{3^0} + \underset{(i=1)}{3^1} + \underset{(i=2)}{3^2}$

$= 1 + 3 + 9$

$= 13$

b) $\sum_{j=2}^{8} 7 = \underset{(j=2)}{7} + \underset{(j=3)}{7} + \dots + \underset{(j=8)}{7}$

$= 7(7)$

terms = last index − first index + 1

$= 49$

Ex: Write in sigma notation:

$\frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots$

$\sum_{n=0}^{\infty} \frac{1}{n+6}$ or $\sum_{n=6}^{\infty} \frac{1}{n}$ or $\sum_{n=1}^{\infty} \frac{1}{n+5}$

3.2 Arithmetic Sequences and Series

0.4, 0.5, 0.6, 0.7, ...

Common difference

5, -15, -25, ...

-10 -10