Quiz Tues 13th Section 2.5

Test Wed 14th
Sections 2.3 - 2.8 (7 Questions)
Practice Problems on website
Formula sheet will be provided

2.7 The Conditional Cont'd

$p \to q$
" p implies q "
" if p then q "

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

← promise is broken

<u>Ex</u>: Is $p \to q$ logically equivalent to $\sim p \vee q$?

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

Identical

Yes. $p \to q \iff \sim p \vee q$

Fact

$$p \to q \iff \sim p \vee q$$

The <u>converse</u> of $p \to q$ is $q \to p$.
The <u>inverse</u> of $p \to q$ is $\sim p \to \sim q$
The <u>contrapositive</u> of $p \to q$ is $\sim q \to \sim p$.

<u>Ex</u>: Build a truth table for
$p \to q$, $q \to p$, $\sim p \to \sim q$, $\sim q \to \sim p$.

p	q	$p \to q$	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$	$\sim q \to \sim p$
0	0	1	1	1	1	1	1
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	1	1	1	0	0	1	1

Identical

Identical

$$p \to q \iff \sim q \to \sim p$$

$$q \to p \iff \sim p \to \sim q$$

Fact

All logically equivalent:
$p \to q$
$\sim q \to \sim p$
$\sim p \vee q$

All logically equivalent:
$q \to p$
$\sim p \to \sim q$
$\sim q \vee p$

Ex: Consider the statement:

$\boxed{\text{" If it's raining then it's cloudy. "}}$ $(p \to q)$

Write the indicated statement.
Is it logically equivalent to the original statement?

a) the converse

$(q \to p)$

If it's cloudy then it's raining.

NO.

b) the inverse

$(\sim p \to \sim q)$

If it's not raining then it's not cloudy.

NO.

c) the contrapositive

$(\sim q \to \sim p)$

If it's not cloudy then it's not raining.

YES

Ex: Write the contrapositive of:

"If I live in Victoria or Vancouver then I live in BC"

Original: $(p \vee q) \longrightarrow r$

Contrapositive: $\sim r \longrightarrow \sim (p \vee q)$

ALTERNATIVELY $\sim r \rightarrow \sim p \wedge \sim q$ (De Morgan's)

If I don't live in BC then I don't live in Victoria and I don't live in Vancouver.

2.8 The Biconditional

Biconditional statement:

$$p \longleftrightarrow q$$

"p if and only if q"

"if p then q and vice versa"

"if and only if p then q"

Means: $p \rightarrow q$ and $q \rightarrow p$.

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Ex: Show that $p \leftrightarrow q$ is logically equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \to q$	$q \to p$	$(p\to q)\wedge(q\to p)$	$p \leftrightarrow q$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

Identical ✓

Ex: The following statement is true:

"Snarks are Boojums if and only if the bellman is incorrect."

Answer Yes, No or Maybe.

a) Snarks are Boojums. Is the bellman correct?

No

b) Snarks are not Boojums. Is the bellman correct?

Yes

c) The bellman is correct. Are Snarks Boojums?

No

d) The bellman is incorrect. Are Snarks Boojums?

Yes

3.1 Sequences and Series

<u>Sequence</u>: ordered list of numbers

A sequence is <u>finite</u> if it has a final term.

A sequence is <u>infinite</u> otherwise.

Quick Ex: $2, 5, 8, 11, \dots, 32$ is a finite sequence

$2, 5, 8, \dots$ is an infinite sequence

<u>Ex</u>: Identify the pattern

a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{256}$

multiply by $\frac{1}{2}$
(divide by 2)

b) $1, 4, 9, 16, \dots$

squares

c) $5, -5, -15, \ldots$

subtract 10

A sequence can be indexed starting at any non-negative integer. Could be written:

$a_0, a_1, a_2, \ldots$

$a_1, a_2, a_3, \ldots$

$a_m, a_{m+1}, a_{m+2}, \ldots$

3 ways to define a sequence:

1) List a few terms.

2) Give a general formula for a_n in terms of n.

3) Give a recursive formula for a_n in terms of previous term(s).

LIST $2, 5, 8, \ldots, 32$

GENERAL FORMULA $a_n = 3n - 1 \quad 1 \le n \le 11$

RECURSIVE FORMULA $\begin{cases} a_1 = 2 \\ a_n = 3 + a_{n-1} \text{ for } 2 \le n \le 11 \end{cases}$

Ex: $a_n = 4n - 1$ for $n \ge 1$

Find a_1, a_2 and a_{100}.

$n=1$: $a_1 = 4(1) - 1$
$= 3$

$n=2$: $a_2 = 4(2) - 1$
$= 7$

$n=100$: $a_{100} = 4(100) - 1$
$= 399$

Ex: $a_n = 2^n + 1$ for $0 \le n \le 3$

a) Write out all the terms.

$a_0 = 2^0 + 1 = 2$
$a_1 = 2^1 + 1 = 3$
$a_2 = 2^2 + 1 = 5$
$a_3 = 2^3 + 1 = 9$

b) How many terms are there?

4

# of terms = (last index) − (first index) + 1

Ex: Give a general formula for:

a) $1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \ldots$

$a_n = \sqrt{n}$ for $n \ge 1$

b) $1, \sqrt{2}, \sqrt{3}, 2, \ldots, \sqrt{10}$

$a_n = \sqrt{n} \qquad 1 \leq n \leq 10$