Quiz Tues Feb 6: Section 2.3

2.5 Laws of Logic Cont'd

Last week:

Identity Laws

Commutative Laws

Associative Laws

Idempotent Laws

$p \wedge p \Longleftrightarrow p$ $\qquad AA = A$

$p \vee p \Longleftrightarrow p$ $\qquad A+A = A$

Quick Ex:

$(\sim p \wedge q) \vee (\sim p \wedge q) \Longleftrightarrow \sim p \wedge q$

$(A+\overline{B})(A+\overline{B}) = A+\overline{B}$

Complement Laws

$\sim\sim p \Longleftrightarrow p$ $\qquad \overline{\overline{A}} = A$

$p \wedge \sim p \Longleftrightarrow 0$ $\qquad A\overline{A} = 0$

$p \vee \sim p \Longleftrightarrow 1$ $\qquad A+\overline{A} = 1$

Quick Ex:

$$(p \vee q) \wedge \sim(p \vee q) \Longleftrightarrow 0$$

$$ABC + \overline{ABC} = 1$$

Ex: Simplify

$$((\sim p \vee 0) \wedge (q \vee \sim q)) \wedge (1 \vee r)$$

$\Longleftrightarrow (\sim p \wedge (q \vee \sim q)) \wedge (1 \vee r)$ Identity

$\Longleftrightarrow (\sim p \wedge 1) \wedge (1 \vee r)$ Complement

$\Longleftrightarrow (\sim p \wedge 1) \wedge 1$ Identity

$\Longleftrightarrow \sim p \wedge 1$ Identity

$\Longleftrightarrow \sim p$ Identity

Ex: Simplify

$$(p \wedge \sim p) \vee (p \vee \sim p)$$

$\Longleftrightarrow 0 \vee (p \vee \sim p)$ Complement

$\Longleftrightarrow 0 \vee 1$ Complement

$\Longleftrightarrow 1$ Identity

Ex: Simplify

$$\sim(p \vee (q \wedge \sim r)) \wedge (p \vee (q \wedge \sim r))$$

$\Longleftrightarrow 0$ Complement

Ex: Simplify

$$A(\overline{B}B) + B(A+\overline{A})$$

$= A(0) + B(A+\overline{A})$ Complement

$= A(0) + B(1)$ Complement

$= 0 + B$ Identity (twice)

$= B$ Identity

(Can do Commutative Law mentally)

Ex: Show that $A\cdot 1 + B\overline{B} = \overline{\overline{A\cdot 1}}$

$A\cdot 1 + B\overline{B} = A + B\overline{B}$ Identity

$= A + 0$ Complement

$= A$ Identity

$= \overline{\overline{A}}$ Complement

$= \overline{\overline{A\cdot 1}}$ Identity

Ex: Show that

$$(p\wedge \sim p)\wedge \sim q \Leftrightarrow p\wedge(q\wedge \sim q)$$

$(p\wedge\sim p)\wedge\sim q \Leftrightarrow 0\wedge\sim q$ Complement

$\Leftrightarrow 0$ Identity

$\Leftrightarrow p \wedge 0$ Identity

$\Leftrightarrow p \wedge (q \wedge \sim q)$ Complement

Alternatively:

$(p \wedge \sim p) \wedge \sim q \Leftrightarrow 0 \wedge \sim q$ Complement

$\Leftrightarrow 0$ Identity

same

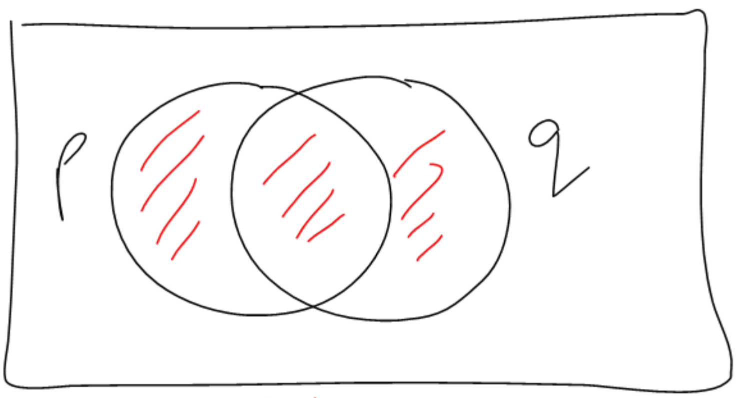
$p \wedge (q \wedge \sim q) \Leftrightarrow p \wedge 0$ Complement

$\Leftrightarrow 0$ Identity

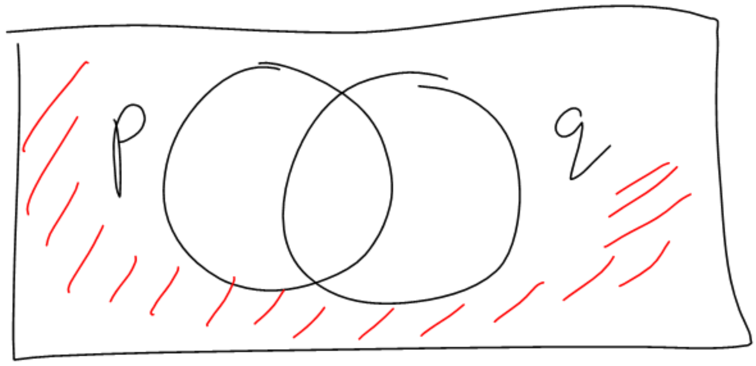
2.6 More Laws of Logic

De Morgan's Laws

$$\sim (p \vee q) \Leftrightarrow \sim p \wedge \sim q$$



$p \vee q$



$$\sim(p \vee q)$$

$$\Longleftrightarrow$$

$$\sim p \wedge \sim q$$