Test 1 Average: 86%

Quiz Tues Feb 6 Section 2.3

Tues Feb 13 Section 2.5

Test 2

Wed Feb 14

Sections 2.3–2.8

2.4 Boolean Expressions and Gate Representations Cont'd

Classical symbols	Boolean symbols	
p, q, r	A, B, C	
$p \vee q$	$A+B$	(OR)
$p \wedge q$	$A \cdot B$, AB	(AND)
$\sim p$	$\overline{A}$	(NOT)
$\Longleftrightarrow$	$=$	(LOGICALLY EQUIVALENT)

Ex: Write as a Boolean expression

$$p \vee (\sim q \wedge r)$$

$$A + \overline{B}C$$

Order of Operations

NOT, then AND, then OR.

Brackets override the order.

(*) The negation bar acts like brackets: do the operation under the negation bar then the negation.

Ex: What is the order of operations?

a) $\overline{A}B + C$ NOT, AND, OR

b) $A + \overline{BC}$ AND, NOT, OR

c) $\overline{A+B}\,C$ OR, NOT, AND

d) $(A+\overline{B})C$ NOT, OR, AND

Ex: Is $\overline{A+B}$ logically equivalent to $\overline{A} + \overline{B}$?

A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	1
1	0	1	0	0	1	1
1	1	1	0	0	0	0

Identical?

No

ASIDE

AND

T and F $\Longleftrightarrow$ F

$1 \cdot 0 = 0$

OR

T or F $\Longleftrightarrow$ T

$1 + 0 = 1$

T or T $\Longleftrightarrow$ T

$1 + 1 = 1$

Boolean algebra is different than real number algebra.

Ex: Simplify $AB + A\overline{B}$

A	B	AB	$\overline{B}$	$A\overline{B}$	$AB + A\overline{B}$
0	0	0	1	0	0
0	1	0	0	0	0
1	0	0	1	1	1
1	1	1	0	0	1

$$AB + A\overline{B} = A$$

"Either A and B or A and not B" means the same thing as A.

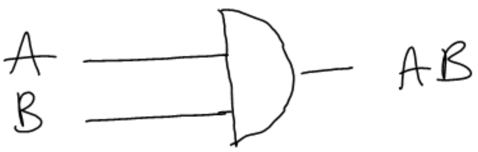
Ex: Build the truth table for

$$A + \overline{B+C}$$

A	B	C	B+C	$\overline{B+C}$	$A+\overline{B+C}$
0	0	0	0	1	1
0	0	1	1	0	0
0	1	0	1	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	1	0	1

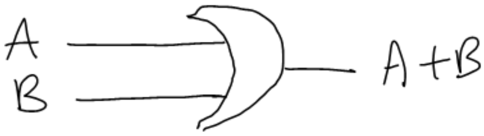
A *gate representation* is a way to visualize logical expressions.

AND



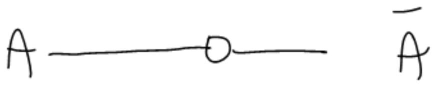
(rigid)

OR

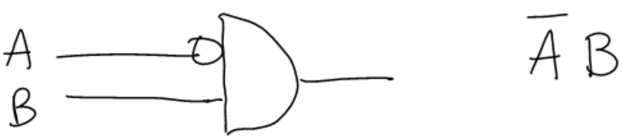


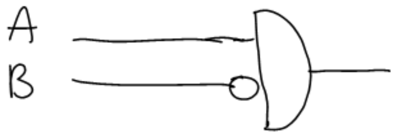
(flexible)

NOT

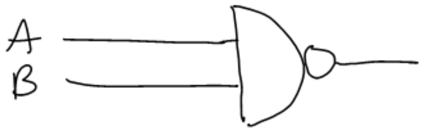


Quick Ex:





$A\overline{B}$



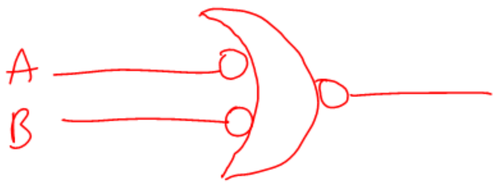
$\overline{AB}$

Ex: Draw the gate representation:

a) $A+\overline{B}$

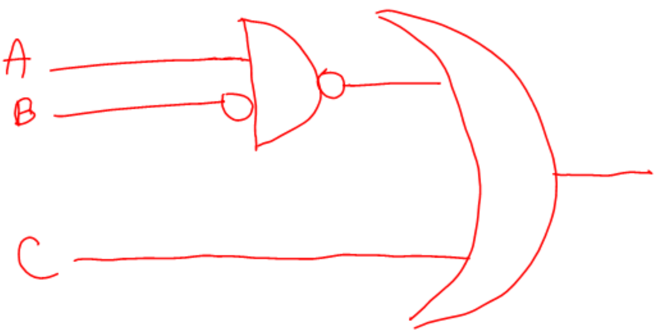
$A+\overline{B}$

b) $\overline{\overline{A}+\overline{B}}$



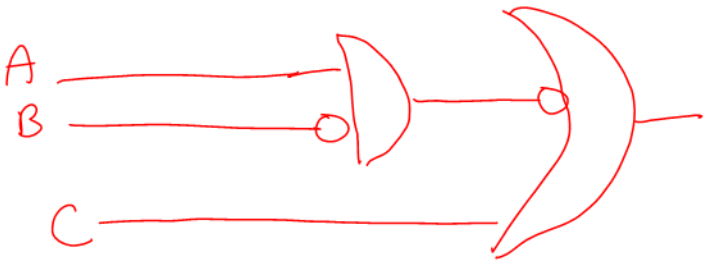
$\overline{\overline{A}+\overline{B}}$

c) $\overline{A\overline{B}}+C$



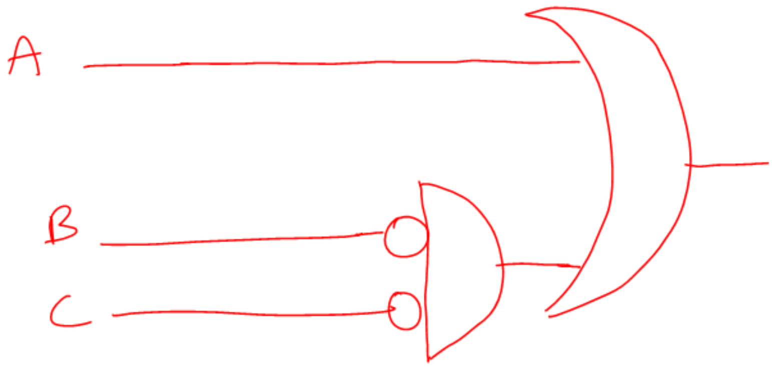
$\overline{A\overline{B}}+C$

Also acceptable:



$\overline{A\overline{B}} + C$

d) $A + \overline{B}\,\overline{C}$



2.5 Laws of Logic

Ex: Show that $p \wedge 1 \iff p$

p	1	$p \wedge 1$
0	1	0
1	1	1

Identical ✓

In Boolean symbols:

$$A \cdot 1 = A$$

There are four "Identity Laws"

Classical Symbols	Boolean Symbols
$p \wedge 1 \iff p$	$A \cdot 1 = A$
$p \wedge 0 \iff 0$	$A \cdot 0 = 0$
$p \vee 1 \iff 1$	$A + 1 = 1$
$p \vee 0 \iff p$	$A + 0 = A$

Ex: Simplify $(q \wedge 1) \vee (p \wedge 0)$

$\iff q \vee (p \wedge 0)$ Identity

$\iff q \vee 0$ Identity

$\iff q$ Identity

Commutative Laws

$p \wedge q \iff q \wedge p$	$AB = BA$
$p \vee q \iff q \vee p$	$A + B = B + A$

Ex: Simplify $0 \wedge p$

$\iff p \wedge 0$ Commutative

$\iff 0$ Identity

(Can do Commutative Law mentally)

Associative Laws

$(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$ $(AB)C = A(BC)$

$(p \vee q) \vee r \iff p \vee (q \vee r)$ $(A+B)+C = A+(B+C)$

Ex: Simplify $(p \wedge 1) \wedge 1$

$\iff p \wedge (1 \wedge 1)$ Associative

$\iff p \wedge 1$ Identity

$\iff p$ Identity

Idempotent Laws

$p \wedge p \iff p$ $AA = A$

$p \vee p \iff p$ $A+A = A$