

## 2.3 Logical Equivalence Cont'd

Two logical expressions are logically equivalent if they have the same sequence of truth values.

Ex: Is  $\sim(p \vee q)$  logically equivalent to  $\sim p \wedge \sim q$ ?

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

↑ Identical? ↑

YES

$$\sim(p \vee q) \iff \sim p \wedge \sim q$$

Ex: Is  $\sim(p \oplus q)$  logically equivalent to  $\sim p \oplus \sim q$ ?

p	q	$p \oplus q$	$\sim(p \oplus q)$	$\sim p$	$\sim q$	$\sim p \oplus \sim q$
0	0	0	1	1	1	0
0	1	1	0	1	0	1
1	0	1	0	0	1	1
1	1	0	1	0	0	0

No

$$\sim(p \oplus q) \not\leftrightarrow \sim p \oplus \sim q$$

### Notation

0 is the expression that is always false.  
 1 is the expression that is always true.

Ex: Simplify  $p \wedge \sim p$  using a truth table.

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

$$p \wedge \sim p \iff 0$$

Ex: Simplify  $p \wedge 1$  using a truth table.

p	1	$p \wedge 1$
0	1	0
1	1	1



$$p \wedge 1 \iff p$$

Ex: Simplify  $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$   
using a truth table.

$p$	$q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge \sim q$	$(\sim p \wedge \sim q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	1	0	0	0	0	0

↑ ↑  
Identical

$$(\sim p \wedge \sim q) \vee (p \wedge \sim q) \iff \sim q$$

## 2.4 Boolean Expressions and Gate Representations

Classical symbols

$p, q, r$

$\sim p$

$p \wedge q$

$p \vee q$

$\iff$

Boolean symbols

$A, B, C$

$\bar{A}$

(NOT)

$AB$  or  $A \cdot B$

(AND)

$A+B$

(OR)

$=$

(LOGICALLY  
EQUIVALENT)

Ex: Write as a Boolean expression:

$$p \wedge (q \vee \sim r)$$

$$A (B + \bar{C})$$