(21) Statement ($p \to q$):
If Bak is out then his keys are in his pocket.

The contrapositive ($\sim q \to \sim p$):
If Bak's keys are not in his pocket then Bak is not out.
☆ Logically equivalent to original statement ☆

a) Maybe
b) Yes
c) No
d) Maybe

(23) a) No

b) Yes

c) Yes

d) No

26 Arithmetic Sequence

$a_1 = 23 \quad d = 8$

$a_n = a_m + (n-m)d$ for $n \geq m$

$m=1$: $a_n = a_1 + (n-1)d$ for $n \geq 1$

$a_n = 23 + (n-1)(8)$ "

$a_n = 23 + 8n - 8$ "

$$\boxed{a_n = 15 + 8n \quad \text{for } n \geq 1}$$

27 $a_1 = 19 \quad d = 6$

a) $a_n = a_m + (n-m)d$

$m=1$: $a_n = a_1 + (n-1)d$

$a_1 = 19$, $d = 6$: $a_n = 19 + (n-1)(6)$

$n = 35$: $a_{35} = 19 + 34(6)$

$a_{35} = 223$

b) From above

$$a_n = 19 + (n-1)(6)$$

$a_n = 1243$:

$$1243 = 19 + (n-1)(6)$$
$$1224 = (n-1)(6)$$
$$204 = n-1$$
$$205 = n$$

$a_{205} = 1243$

29 $4, -8, 16, -32, \ldots$

$\times(-2)$ $\times(-2)$ $\times(-2)$

$r = \frac{-8}{4} = -2$

Geometric Sequence

$a_1 = 4$

$r = -2$

$$\begin{cases} a_1 = 4 \\ a_n = -2a_{n-1} \end{cases} \quad \text{for } n \geq 2$$

next term

previous term

(31) $-6 + 3 - \frac{3}{2} + \frac{3}{4} - \ldots$

Geometric Series

$a_1 = -6$

$r = \frac{3}{-6} = -\frac{1}{2}$

a) $S_k = \frac{a_m(1-r^k)}{1-r}$

$k=8$: $S_8 = \frac{a_m(1-r^8)}{1-r}$

$a_m = -6$
$r = -\frac{1}{2}$: $S_8 = \frac{-6\left(1-\left(-\frac{1}{2}\right)^8\right)}{\left(1+\frac{1}{2}\right)}$

≈ -3.98

b) $-1 < r < 1$ ✓

$$S_\infty = \frac{a_n}{1-r}$$

$$= \frac{-6}{\left(1+\frac{1}{2}\right)}$$

$$= -4$$

33 a) $O(n)$

b) $O(n^2)$

c) $O(1)$

d) $O(n!)$

e) $O(2^n)$

f) $O(n!)$

g) $O(\log n)$

h) $6n(20+3\log n)$

$= 120n + 18n\log n$

$O(n\log n)$

i) $O(n)$