Quiz Tues March 26
Section 6.3

8.1 Counting Techniques Cont'd

Ex: How many 6-digit PINs:

a) don't have repeated digits?

$$\boxed{10} \times \boxed{9} \times \boxed{8} \times \boxed{7} \times \boxed{6} \times \boxed{5}$$

# of options for 1st digit

2nd digit

$= 151,200$

b) have at least one "3"?

Complement Rule

total # of PINs $= 10 \times 10 \times \ldots \times 10$

$= 10^6$

# of PINs with no 3's

$$\underset{\text{not } 3}{\boxed{9}} \times \underset{\text{not } 3}{\boxed{9}} \times \boxed{9} \times \boxed{9} \times \boxed{9} \times \underset{\text{not } 3}{\boxed{9}}$$

$= 9^6$

# of PIN's with at least one "3"

$= 10^6 - 9^6$

$= 468,559$

c) start with 1 or end with 2?

Inclusion-Exclusion Rule

$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

$n(\text{start } 1) = 1 \times 10 \times 10 \cdots \times 10 = 10^5$

$n(\text{end } 2) = 10 \times 10 \cdots \times 10 \times 1 = 10^5$

$n(\text{start } 1 \text{ and end } 2) = 1 \times 10 \times 10 \cdots \times 10 \times 1$

$= 10^4$

$n(\text{start } 1 \text{ or end } 2) = 10^5 + 10^5 - 10^4$
$= 190{,}000$

d) start with 1 or 2?

no overlap

Don't need Inclusion-Exclusion.

$\boxed{2} \times \boxed{10} \times \boxed{10} \times \cdots \times \boxed{10}$

$= 2 \times 10^5$

$= 200{,}000$

8.2 Classical Probability

If all outcomes are equally likely then the probability of E happening is:

$$P(E) = \frac{n(E)}{n_{total}}$$

\# of ways E can happen

Probability of E happening

total # of possible outcomes

<u>Ex</u>: Roll a fair die. Find the probability that the roll is greater than 2.

<u>fair</u>: all outcomes are equally likely

We say one <u>die</u>, two <u>dice</u>.

All possible outcomes: 1, 2, 3, 4, 5, 6

$$n_{total} = 6$$

$$n(E) = 4$$

$$P(E) = \frac{4}{6}$$

or $P(\text{roll is greater than } 2) = \frac{4}{6}$

$$\boxed{\begin{array}{l} \text{FACT} \\ P(A) = 1 - P(\text{not } A) \\ P(A) = 1 - P(\bar{A}) \end{array}}$$

Why? $P(A) + P(\text{not } A) = 100\%$

$$P(A) + P(\text{not } A) = 1$$

$$P(A) = 1 - P(\text{not } A)$$

Ex: Roll two fair 4-sided dice.

Find the probability that:

a) the rolls sum to 3 or less

(red) Die #1 \ Die #2 (blue)

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$$\frac{3}{16}$$

b) the rolls sum to greater than 3

$$1 - P(\text{rolls sum to at most } 3)$$

$$= 1 - \frac{3}{16}$$

$$= \frac{16}{16} - \frac{3}{16}$$

$$= \frac{13}{16}$$

c) the rolls sum to 5

$$\frac{4}{16}$$

d) at least one roll is a 2.

$$\frac{7}{16}$$