8.1 Counting Techniques Cont'd

Ex: How many 5-character case-sensitive alphanumeric passwords are there:

a) in total?

symbols: $0, 1, \ldots, 9, a, b, \ldots, z, A, B, \ldots, Z$

# of symbols $= 10 + 26 + 26$
$= 62$

$$\boxed{62} \times \boxed{62} \times \boxed{62} \times \boxed{62} \times \boxed{62}$$

# of options for 1st symbol; 2nd; 3rd; 4th; 5th

or 62^5

or $916,132,832$

b) that contain at least one letter and at least one number?

# of passwords with no letters

$= \boxed{10} \times \boxed{10} \times \boxed{10} \times \boxed{10} \times \boxed{10}$

0-9 ; 0-9

$= 10^5$

# of passwords with no numbers

$= \boxed{52} \times \boxed{52} \times \boxed{52} \times \boxed{52} \times \boxed{52}$

1st symbol ... 5th symbol

$= 52^5$

# of passwords that contain at least 1 letter and at least 1 number

$= 62^5 - 10^5 - 52^5$

$= 535,828,800$

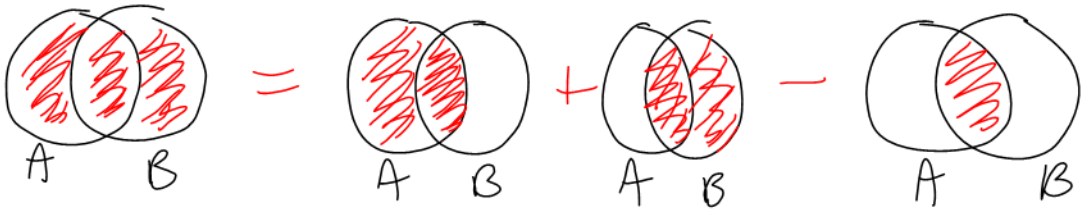
Notation

$n(A)$ = # of ways that A can happen

Inclusion-Exclusion Rule

$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$



Ex: Count how many integers from 1 to 15 are divisible by 3 or 5:

a) directly

~~1~~ ~~2~~ (3) ~~4~~ (5) (6) ~~7~~ ~~8~~ (9) (10) ~~11~~ (12) ~~13~~ ~~14~~ (15)

7

b) by using Inclusion-Exclusion

$n(\text{divisible by } 3) = 5$ (3, 6, 9, 12, 15)

$n(\text{divisible by } 5) = 3$ (5, 10, 15)

$n(\text{divisible by 3 and 5}) = 1$ (15)

$n(\text{divisible by 3 or 5}) = n(\text{divisible by 3}) + n(\text{divisible by 5}) - n(\text{divisible by 3 and 5})$

$= 5 + 3 - 1$

$= 7$ ✓

Ex: How many 4-digit PINs:

a) start with 9?

$1 \times 10 \times 10 \times 10 = 1000$

9 0-9 0-9 0-9

b) end with 4?

$10 \times 10 \times 10 \times 1 = 1000$

0-9 0-9 0-9 4

c) start with 9 and end with 4?

$$\boxed{1}_{9} \times \boxed{10}_{0\text{-}9} \times \boxed{10}_{0\text{-}9} \times \boxed{1}_{4} = 100$$

d) start with 9 or end with 4?

Inclusion-Exclusion

$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

$n(\text{start } 9 \text{ or end } 4) = n(\text{start } 9) + n(\text{end } 4) - n(\text{start } 9 \text{ and end } 4)$

$= 1000 + 1000 - 100$

$= 1900$

e) start with 4 or 9?

can't both happen

Don't need Inclusion-Exclusion

$$\boxed{2}_{4 \text{ or } 9} \times \boxed{10} \times \boxed{10} \times \boxed{10} = 2000$$