Statistics Suggested HW is on website
Quiz Tues March 19 Section 6.1

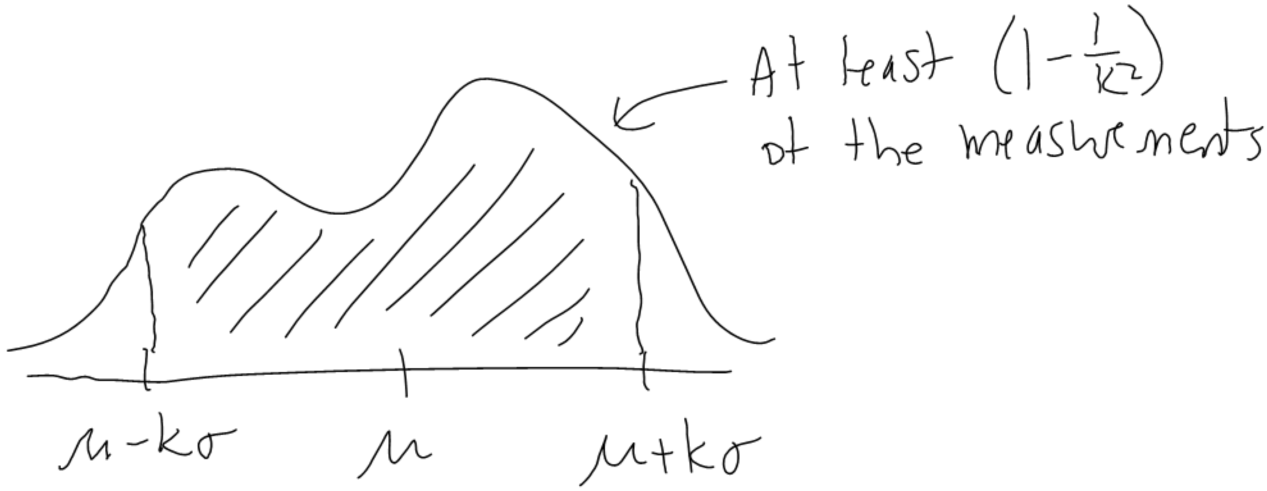
σ = population standard deviation
s = sample "

$$\sigma \approx s$$

6.3 Tchebysheff and Empirical Rules

Tchebysheff's Rule:

For any data set:
At least $\left(1-\frac{1}{k^2}\right)$ of the measurements fall within $\mu - k\sigma \le x \le \mu + k\sigma$, where k is any real number > 1.

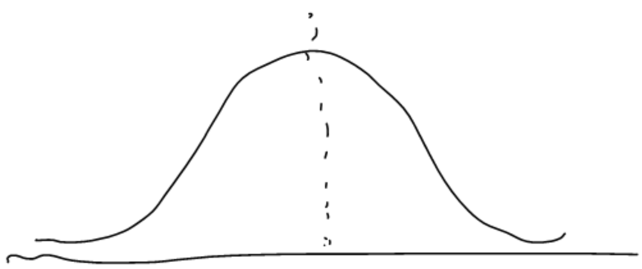


Ex: A data set has $\mu = 10$ and $\sigma = 2$.
Fill in the table.

k	$\mu - k\sigma$	$\mu + k\sigma$	$1 - \frac{1}{k^2}$	Conclusion
2	6	14	0.75 = 75%	At least 75% of measurements fall in $6 \le x \le 14$
3	4	16	0.89 = 89%	At least 89% of measurements fall in $4 \le x \le 16$
4	2	18	0.94 = 94%	At least 94% of measurements fall in $2 \le x \le 18$

A mound-shaped (or bell-shaped) data set is unimodal and symmetrical.

(1 peak)



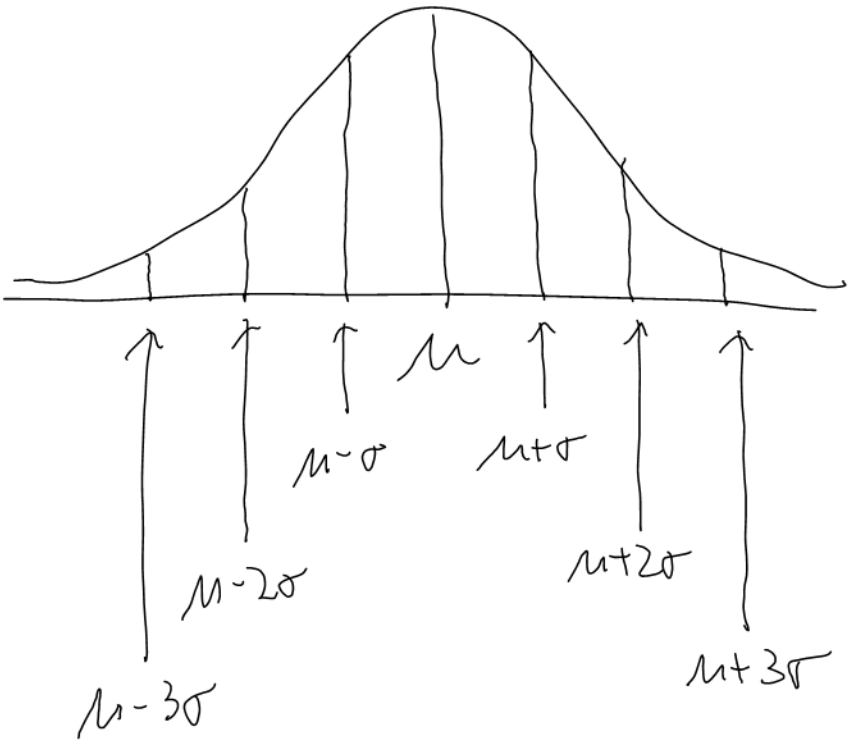
The Empirical Rule

If the data is approximately mound-shaped then

$\mu - \sigma \le x \le \mu + \sigma$ contains approx. 68% of the data

$\mu - 2\sigma \le x \le \mu + 2\sigma$ " " 95% "

$\mu - 3\sigma \le x \le \mu + 3\sigma$ " " 99.7% "



68%

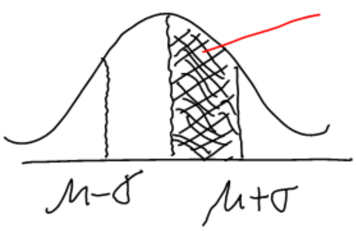
95%

99.7%

Know these 3 percentages.

<u>Ex</u>: Given a <u>mound-shaped</u> data set, estimate the % of data in the shaded region.

a)



$\frac{68\%}{2} = 34\%$ or 0.34

b)

$\frac{95\%}{2} = 47.5\%$ or 0.475

c)

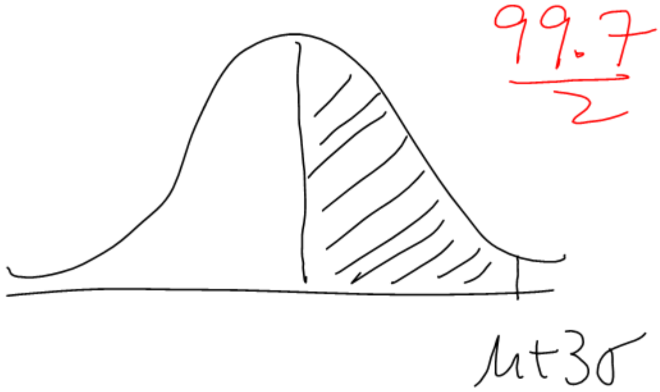
50%

d)

$\frac{95\%}{2} = 47.5\%$

$50\% - 47.5\%$
$= 2.5\%$

e)



$\frac{99.7\%}{2} = 49.85\%$

Ex: At a software company, hours worked last week are mound-shaped with $\mu = 42$ and $\sigma = 2$. What % of employees worked less than 36 hours last week?

$36 = \mu + k\sigma$

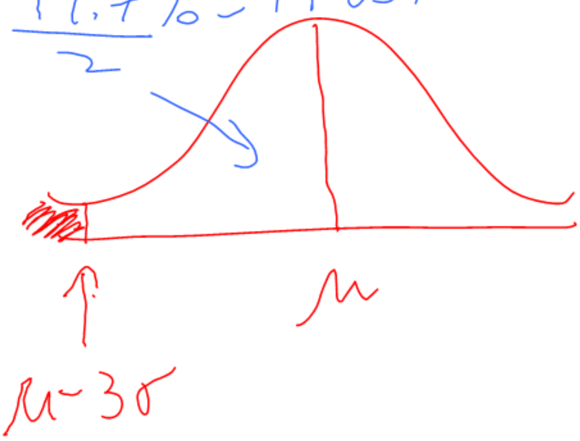
$36 = 42 + k(2)$

$-6 = k(2)$

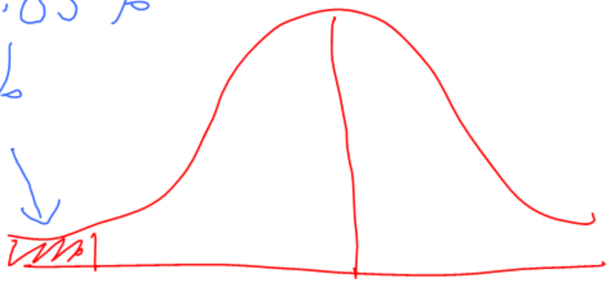
$-3 = k$

$$\boxed{36 = \mu - 3\sigma}$$

$\frac{99.7\%}{2} = 49.85\%$



$50\% - 49.85\%$
$= 0.15\%$



$\mu - 3\sigma$

$\boxed{\text{Approximately } 0.15\%}$

6.4 Measures of Relative Standing

Recall: μ = population mean

$\bar{x}$ = sample mean

σ = population standard deviation

s = sample standard deviation

The z-score is $z = \dfrac{x-\mu}{\sigma}$,

where x is a measurement.

If working with a sample, $z = \dfrac{x-\bar{x}}{s}$

Ex: You run a race and your time is 70 mins. The average time was 60 mins, with a standard deviation of 5 mins. Find the z-score of your time.

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{70-60}{5}$$

$$= 2$$

Your time was 2 standard deviations <u>above</u> the mean.

Ex: Same question, but your time was 45 mins.

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{45-60}{5}$$

$= -3$

Your time was 3 standard deviations <u>below</u> the mean.

FACT

A z-score is the number of standard deviations above or below the mean.

<u>Ex</u>: Find the z-score of:

a) $x = \mu$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{\mu - \mu}{\sigma}$$

$$= \frac{0}{\sigma}$$

$$= 0$$

b) $x = \mu + 1.5\sigma$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{\mu + 1.5\sigma - \mu}{\sigma}$$

$$= \frac{1.5\sigma}{\sigma}$$

$$= 1.5$$

c) $x = \mu - 3\sigma$

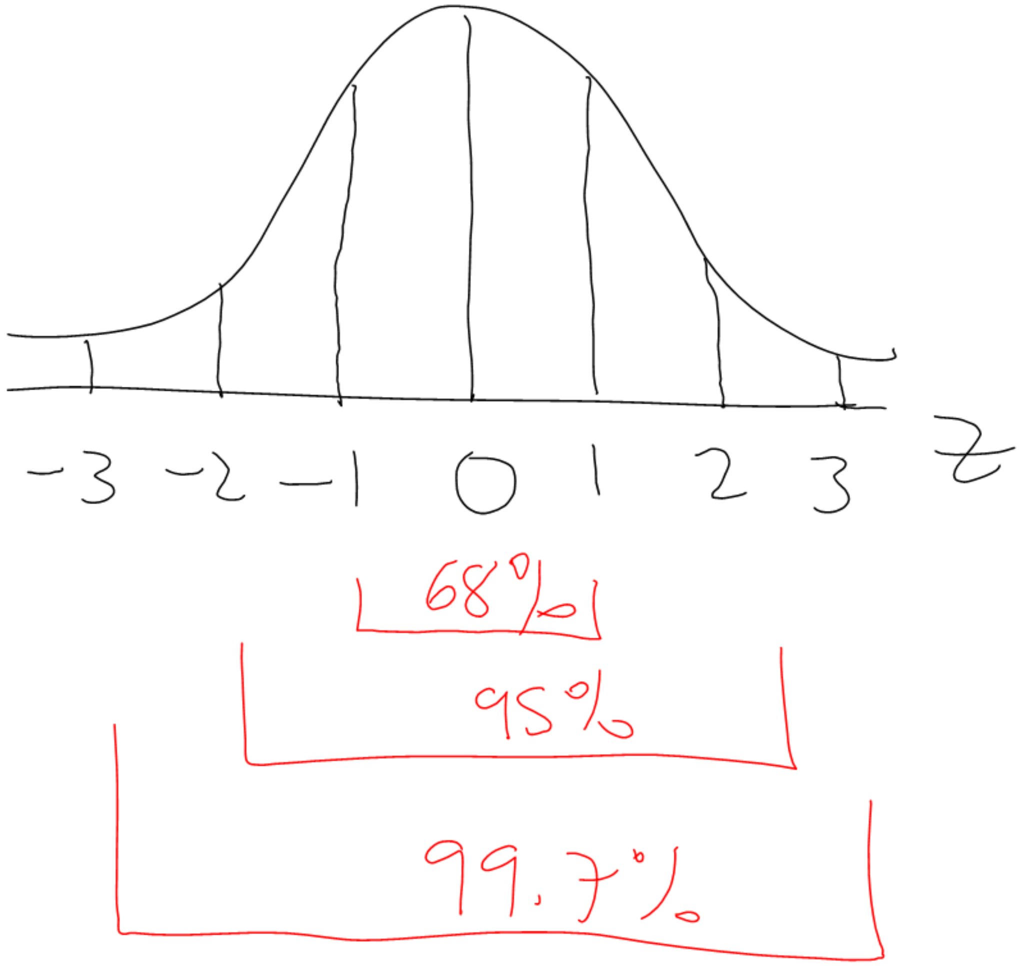
$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{\mu - 3\sigma - \mu}{\sigma}$$

$$= \frac{-3\sigma}{\sigma}$$

$$= -3$$

Ex: Describe the Empirical Rule in terms of z-scores.



FACT

z-scores bigger than 3 or less than -3 are outliers.

Ex: A student wrote two tests.

	Course A	Course B
Student's Mark	74.5	90
Average Mark	70	78
Standard Deviation	1.5	5

In which course did the student do best relative to the class?

Course A: $z = \frac{x-\mu}{\sigma} = \frac{74.5-70}{1.5} = 3$

Course B: $z = \frac{x-\mu}{\sigma} = \frac{90-78}{5} = 2.4$

$\boxed{\text{Course A}}$

6.3 #

37. **Cereal Potassium per Serving** A survey of a number of the leading brands of cereal shows that the mean content of potassium per serving is 95 milligrams, and the standard deviation is 2 milligrams. Find the range in which at least 88.89% of the data will fall. Use Chebyshev's theorem.

$\mu = 95$

$\sigma = 2$

$1 - \frac{1}{k^2} = 0.8889$

$1 - 0.8889 = \frac{1}{k^2}$

$0.1111 = \frac{1}{k^2}$

Take reciprocals: $\frac{1}{0.1111} = k^2$

$9 = k^2$

$3 = k$

$\mu - k\sigma$
$= 95 - 3(2)$
$= 89$

$\mu + k\sigma$
$= 95 + 3(2)$
$= 101$

$$\boxed{89 \le x \le 101}$$