

Test Review

Ex: Write a recursive formula for:

a) $a_n = 7n$ for $4 \leq n \leq 12$

$$28, 35, 42, \dots, 84$$

arithmetic sequence $d=7$

$$\begin{cases} a_4 = 28 \\ a_n = a_{n-1} + 7 \quad 5 \leq n \leq 12 \end{cases}$$

b) $a_n = 3^n$ for $2 \leq n \leq 5$

$$3^2, 3^3, 3^4, 3^5$$

$$\begin{array}{c} \xrightarrow{\quad} \xrightarrow{\quad} \xrightarrow{\quad} \\ \times 3 \quad \times 3 \quad \times 3 \end{array}$$

geometric sequence $r=3$

$$\begin{cases} a_2 = 3^2 \\ a_n = 3a_{n-1} \quad 3 \leq n \leq 5 \end{cases}$$

Ex: Find:

a) $\sum_{n=4}^{11} 2^n$

$$= 2^4 + 2^5 + 2^6 + \dots + 2^{11}$$

$\xrightarrow{\times 2} \quad \xrightarrow{\times 2}$

geometric series $r=2$

$$\begin{aligned} k &= \# \text{ of terms} \\ &= 11 - 4 + 1 \\ &= 8 \end{aligned}$$

$$S_k = \frac{a_n (1 - r^k)}{1 - r}$$

$$\begin{aligned} k=8: \quad S_8 &= \frac{a_n (1 - r^8)}{1 - r} \\ &= \frac{2^4 (1 - 2^8)}{(1 - 2)} \\ &= 4080 \end{aligned}$$

b) $\sum_{n=4}^{11} 9n$

$$= 36 + 45 + 54 + \dots + 99$$

$\xrightarrow{+9} \quad \xrightarrow{+9}$

arithmetic series $d=9$

$$S_k = \frac{k}{2} (a_m + a_n)$$

$$\begin{aligned}
 k &= \# \text{ of terms} \\
 &= 11 - 4 + 1 \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 k=8: \quad S_8 &= \frac{8}{2} (a_m + a_n) \\
 &= \frac{8}{2} (36 + 99) \\
 &= 540
 \end{aligned}$$

Ex: Given the following arithmetic sequence:

$$\begin{cases} a_1 = 7 \\ a_n = 4 + a_{n-1} \quad \text{for } n \geq 2 \end{cases}$$

Find n so that $a_n = 427$.

$$d = 4$$

$$a_n = a_m + (n-m)d$$

$$d=4: \quad a_n = a_m + (n-m)(4)$$

$$m=1: \quad a_n = a_1 + (n-1)(4)$$

$$a_n = 427: \quad 427 = 7 + (n-1)(4)$$

$$a_1 = 7 \quad 420 = (n-1)(4)$$

$$105 = n-1$$

$$106 = n$$

$$\begin{aligned}
 d &= a_2 - a_1 \\
 &= a_8 - a_7
 \end{aligned}$$

$$\begin{aligned}
 r &= \frac{a_2}{a_1} \\
 &= \frac{a_3}{a_2}
 \end{aligned}$$